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The Effect of Mis-Estimating Correlation on Value-At-Risk*

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Abstract

This paper examines the systematic relationship between correlation mis-estimation and the corresponding Value-at-Risk (VaR) mis-calculation. To this end, first a semi-parametric approach, and then a parametric approach is developed. Various linear and non-linear portfolios are considered, as well as variance-covariance and Monte-Carlo simulation methods are employed. We find that the VaR error increases significantly as the correlation error increases, particularly in the case of well-diversified linear portfolios. In the case of option portfolios, this effect is more pronounced for short-maturity, in-the-money options. The use of MC simulation to calculate VaR magnifies the correlation bias effect. Our results have important implications for measuring market risk accurately.

JEL Classification: C15, G10, G13, G21

Keywords: Value-at-Risk, Correlation, Correlation Mis-Estimation, Monte Carlo Simulation, Variance-Covariance Methods, Model Error.

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1 Introduction

In recent years, Value-at-Risk (VaR) has become the standard method to measure the market risk of a portfolio of financial assets. VaR is an estimate of the maximum potential loss to be expected with a certain probability over a given time period. This measure of risk is widely used by practitioners and regulators because of its conceptual simplicity and flexibility. It summarizes all the possible market risks in a single number, and it can take into account various kinds of exposures and risks. However, VaR calculation is subject to various sources of model error such as the model specification, the used data set, and the method employed for the parameters estimation [see Beder (1995), Crouhy, Galai and Mark (1998), and Figlewski (1999)]. The objective of this study is to investigate the relationship between mis-estimating correlation and the induced VaR model error. Understanding this relationship is a necessary prerequisite for measuring market risk accurately.

Correlation is one of the parameters that needs to be estimated so as to implement any parametric VaR model. Any bias in the measurement of correlation between the portfolio assets will yield an erroneously calculated VaR. Despite the plethora of correlation estimators proposed in the literature, the correlation mis-estimation is inevitable[1]. More importantly, in periods of market stress, where VaR is supposed to be used for, the correlations deviate significantly from the estimated values [see Boyer, Gibson and Loretan (1999), Rebonato and Jäckel (2000), and Bhansali and Wise (2001) for a discussion of cases where the statistical techniques can not provide accurate estimates of correlation].

[1] Traditionally, correlation has been modeled as a constant and unconditional variable. However, the empirical evidence [see Kaplanis (1988), and Longin and Solnik (1995) among others] shows that the stability of correlation is a debatable assumption. The Risk Metrics™ Group of J.P. Morgan (1996) proposed the exponentially weighted moving average correlation estimator so as to take into account the time-variability of correlation. In addition, several multivariate GARCH volatility models have been developed in the literature [see Bollerslev, Engle and Wooldridge (1988), Engle and Kroner (1995), Alexander (2001), Tse and Tsui (2002), Engle (2002)]. As an alternative to historical measures of correlation, a number of researchers [see among others Siegel (1997), Campa and Chang (1998), and Walter and Lopez (2000a)] have derived implied correlations from currency options. For a comprehensive review of existing correlation models, see also Alexander (1998).

A number of studies [e.g., Beder (1995), Alexander and Leigh (1997), Davé and Stahl (1997), Jackson, Maude and Perraudin (1997), and Walter and Lopez (2000b)] compare VaR models that use different multivariate volatility models. Their results indicate that different correlation estimators yield significantly different VaR figures. However, the evidence about which is the best correlation estimator within a VaR framework is far from being conclusive.

This paper takes a different route. Accepting that any correlation model estimates the (unknown) 'true' correlation with some error, the question "Does it matter which (false) correlation estimator shall we use for VaR purposes?" arises. We address this question by examining whether there is a systematic relationship between correlation mis-estimation and the induced VaR mis-calculation. This is important for effective risk management. For instance, in the case that there is such a relationship, the VaR model error issue (due to correlation mis-estimation) becomes important; the risk manager should look for the method that estimates correlation with the lowest possible error. On the other hand, if no such systematic relationship exists, the risk manager can switch between the existing correlation estimators without affecting in a significant way his (already) mis-calculated VaR. Then, he can focus on identifying the effect of other sources of model error on his VaR calculation.

First, we examine *semi-parametrically* the "correlation mis-estimation" effect on VaR by considering a general class of unbiased correlation estimators. This is a general approach that makes almost no assumptions about the type of the correlation models used. The only assumption made refers to the distribution (normal) of the correlation errors. Next, we study *parametrically* the effect of mis-estimating correlation on VaR by using three popular correlation estimators: the moving average, the exponentially weighted moving average, and the GARCH correlation model. In both approaches, the relationship between VaR and correlation errors is examined for linear and non-linear portfolios. We calculate VaR by employing two different methods: the variance-covariance and the Monte Carlo (MC) simulation methods. This allows us to investigate also whether the VaR model error arising from mis-estimating correlation depends on the type of portfolio, and the method used to calculate VaR.

We find similar results for both the semi-parametric and the parametric analysis: the VaR mis-estimation increases significantly, as the correlation error increases. Moreover, the VaR of well-diversified linear portfolios is more exposed to mis-estimating correlation. For option portfolios, the magnitude of the VaR error sensitivity depends on the moneyness level, the time-to-maturity and the (unknown) 'true' correlation. In particular, the correlation bias effect is more pronounced for short-maturity in-the-money options. Furthermore, we find that the sensitivity of the VaR induced bias to correlation errors depends on the method that is used to calculate VaR; the results indicate that the calculated via MC simulation VaR is more sensitive to correlation mis-estimation. These results hold for both linear and non-linear portfolios, and they have important implications for risk management.

The rest of the paper is organized as follows: Section 2 describes the variance-covariance and the MC simulation methods we employ to calculate VaR. Section 3 describes the methodology used to investigate the relationship between VaR error and correlation mis-estimation. In Sections 4 and 5, the sensitivity of VaR error due to correlation mis-estimation is measured by developing and applying the semi-parametric and parametric approaches, respectively. Results are shown for various linear and non-linear portfolios. Section 6 concludes, and discusses the implications of the research.

2 Methods to Calculate Value-at-Risk

VaR is defined as the largest loss that a portfolio is likely to suffer if it is left unmanaged during a fixed holding period. More precisely, for a given time horizon T and a confidence interval α, the α%

T-period VaR is the portfolio loss x in market value over the time horizon T that is not expected to be exceeded with probability α, i.e.,

$$Prob(\Delta_T \Pi_t \leq x) = \alpha \tag{1}$$

where $\Delta_T \Pi_t = \Pi_{t+T} - \Pi_t$ is the change in the portfolio value (Profit/Loss, P/L hereafter) over the holding period T. The portfolio value Π_t at time t is a function of the nominal amount W_i invested in the ith asset, and the price P_{it} of the ith asset at time t, i.e.

$$\Pi_t = f(W_i, P_{it}), \quad i=1,\ldots,n. \tag{2}$$

where n is the number of assets in the portfolio.

Several methods have been developed to calculate VaR. These are classified within three main approaches: the variance-covariance, the historical simulation, and the MC simulation approach [see Jorion (1996), Duffie and Pan (1997), and Linsmeier and Pearson (2000) for a detailed description of these methods]. For the purposes of our study, we calculate VaR using the variance-covariance and the MC simulation methods since only these require estimates of the volatilities and the pair-wise correlations of the portfolio assets. Next, we describe briefly the Delta-Normal (DN) and Fallon's (1996) Delta-Gamma (DG) variance-covariance methods, as well as the MC simulation method.

2.1 Variance-Covariance Methods: The Delta-Normal Approach

In the case that the instruments in a portfolio depend linearly on the market factors, the DN risk measurement model developed by Garbade (1986) can be used. Assuming that the portfolio returns follow a multivariate normal distribution and that they are serially independent, the DN portfolio VaR(α, T, $\boldsymbol{\rho}$) that corresponds to a confidence interval α and holding period T is given by:

$$VaR(\alpha,T,\boldsymbol{\rho}) = z_\alpha \sigma_\Pi \sqrt{T} - \mu_\Pi \tag{3}$$

where z_α is the inverse normal cumulative probability function corresponding to probability α, σ_Π is the portfolio's standard deviation, μ_Π is the expected portfolio P/L, and $\boldsymbol{\rho}$ is a $\left(\frac{n!}{2(n-2)!} \times 1\right)$ vector of the pair-wise correlation coefficients ρ_{ij}. The portfolio standard deviation is calculated by:

$$\sigma_\Pi = \sqrt{\sum_{i=1}^{n} W_i^2 \sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j \neq i}^{n} W_i W_j \sigma_i \sigma_j \rho_{ij}} \tag{4}$$

where σ_i is the standard deviation of the ith asset returns.

2.2 Variance-Covariance Methods: The Delta-Gamma Approach

The DN approach is inappropriate for portfolios consisting of non-linear instruments, i.e. instruments whose prices are non-linear functions of the market variables (e.g., options). In such a case, a DG approach could be used. This improves on the DN method by using a second order Taylor series expansion so as to capture non-linearity. For the purposes of our study, we use Fallon's (1996) DG method that can be described as follows.

Assume a portfolio consisting of n options written on n underlying (non-dividend paying) assets, respectively. To calculate the option portfolio VaR, the underlying asset prices are considered as the factors affecting the portfolio value (market factors). In a Black – Scholes (1973) setup, the price V_{it} at time t of the option written on the ith asset is a function of the underlying asset price P_{it} at time t, the option strike price X_i, the option's time-to-maturity $(T_i - t)$, the risk-free rate r_f, and the standard deviation σ_i of the ith asset, i.e.

$$V_{it} = V(P_{it}, X_i, T_i - t, r_f, \sigma_i). \tag{5}$$

The change in the value of the ith option is approximated by a second-order Taylor series expansion, i.e.

$$\Delta V_i \cong \Delta_i (\Delta P_i) + \tfrac{1}{2} \Gamma_i (\Delta P_i)^2 + \Theta_i \Delta t , \tag{6}$$

where Δ_i, Γ_i and Θ_i are the delta, gamma, and theta of the option on asset i, respectively, $\Delta V_i = V_{i,t+T} - V_{i,t}$, $\Delta P_i = P_{i,t+T} - P_{i,t}$, and $\Delta t = (t + T) - t = T$ is the VaR horizon.

The return of the ith asset is defined as $R_i = \Delta P_i / P_i$, where R_i is assumed to follow a normal distribution with zero mean and variance σ_i^2. Then, the change in the portfolio value is a function of the change ΔV_i in the value of each option:

$$\Delta \Pi = \sum_{i=1}^{n} \Delta V_i \cong \boldsymbol{\Delta}^T \boldsymbol{R} + \frac{1}{2} \boldsymbol{R}^T \boldsymbol{\Gamma} \boldsymbol{R} + \sum_{i=1}^{n} \Theta_i \Delta t \ , \tag{7}$$

where $\boldsymbol{\Delta}^T$ denotes the $(1\mathrm{x}n)$ transposed vector with elements $\Delta_i P_i$, $\boldsymbol{R}$ denotes the $(n\mathrm{x}1)$ vector of asset returns, and $\boldsymbol{\Gamma}$ denotes the $(n\mathrm{x}n)$ diagonal matrix with diagonal elements, $\Gamma_i P_i$.

The a percentile of the $\Delta\Pi$ distribution is approximated by the Cornish-Fisher (1937) expansion that uses the first four (known) moments k_1, k_2, k_3 and k_4 of the (non-normal) $\Delta\Pi$ distribution, i.e.:

$$VaR(\alpha,T) \cong -k_1 - w_\alpha * \sqrt{k_2}\ . \tag{8}$$

where

$$w_\alpha = z_\alpha + \frac{1}{6}(z_\alpha^2 - 1) * k_3 + \frac{1}{24}(z_\alpha^3 - 3z_\alpha) * k_4 - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha) * k_3^2\ ,$$

$$k_1 = \frac{1}{2} trace[\mathbf{\Gamma\Sigma}] + \sum_{i=1}^{n} \Theta_i \Delta t\ ,$$

$$k_2 = \Delta^T \Sigma \Delta + ½\ trace\ [(\Gamma \Sigma)^2],$$

$$k_3 = (3\ \Delta^T \Sigma\ [\Gamma \Sigma]\ \Delta + trace[\Gamma \Sigma]^3)/(k_2)^{3/2},$$

$$k_4 = (12\ \Delta^T \Sigma\ [\Gamma \Sigma]^2\ \Delta + 3\ trace[\Gamma \Sigma]^4)/k_2,$$

where $\mathbf{\Sigma}$ is the variance-covariance matrix of the portfolio asset returns, and 'trace' is an operator that sums the diagonal elements of a matrix.

2.3 Monte-Carlo Simulation

MC simulation is recognized by researchers as the most powerful method to compute VaR. In contrast to the variance-covariance approach, MC simulation can take into account nonlinear positions, time-variation in volatility, fat tails, and a wide range of exposures and risks. On the other hand, the main drawback of this method is that it becomes computationally expensive for portfolios containing a large number of assets.

To calculate a portfolio's VaR, we adopt the standard assumption that the price of the *i*th asset follows a Geometric Brownian Motion, i.e.

$$P_{i,t+\Delta t} = P_{it} \exp\left[\left(\mu_i - \frac{\sigma_i^2}{2}\right)\Delta t + \sigma_i \; \xi_i \sqrt{\Delta t}\right], \tag{9}$$

where μ_i is the instantaneous drift of asset i and ξ_i is a normally distributed random variable with mean zero and variance one.

Using equation (9), a path for the ith asset is simulated up to time T. The simulated portfolio market value Π_T at the end of the holding period T can be calculated using the portfolio value function [equation (2)]. To calculate Π_T in the case of option portfolios, an option pricing model needs to be used so as to calculate each option price V_{iT} at time T. This procedure is repeated a large number of times (simulation runs) to create a distribution of Π_T. In our study, we use 30,000 simulation runs and $N = 100$ time steps. The time interval Δt is set equal to 0.01 (T/N=1/100). Let Π_t be the (known) portfolio value at time t. We subtract Π_t from each simulated value Π_T to build the portfolio P/L probability distribution. The VaR(α,T) of the portfolio is the α percentile of this probability distribution. In the case that the portfolio asset prices are uncorrelated, then the random number generation of ξ_i's can be done independently for each asset. However, in general the asset prices are correlated. To create correlated random numbers, the Cholesky decomposition is used [see Jorion (1996) and J.P. Morgan RiskMetrics (1996) for a detailed description of the Cholesky decomposition].

3 Correlation Mis-Estimation and Value-at-Risk: The Methodology

Volatility and correlation estimates are required for VaR calculation using either variance-covariance or MC simulation methods. In practice, these estimates are subject to errors. We focus on measuring the sensitivity of VaR calculation to errors in the estimated pair-wise correlations. Each one of the standard deviations of the portfolio assets are assumed to have been estimated accurately. The general methodology can be described as follows.

Let a class of K correlation estimators exist, and $\hat{\rho}_{ij,k}$ be the pair-wise correlation estimate between assets i and j based on the kth correlation estimator, $k = 1,\ldots,K$. Hence,

$$\hat{\rho}_{ij,k} = \rho_{ij} + \varepsilon_{ij,k}, \quad k = 1,\ldots,K, \tag{10}$$

where ρ_{ij} is the 'true' pair-wise correlation between assets i and j, and $\varepsilon_{ij,k}$ is the pair-wise **correlation estimation error** for any given estimator k.

Let $VaR(\alpha, T, \boldsymbol{\rho})$ be the 'true' portfolio's VaR, corresponding to probability α and holding period T, where $\boldsymbol{\rho}$ is the $\left(\frac{n!}{2(n-2)!} \times 1\right)$ vector of the 'true' pair-wise correlation coefficients ρ_{ij}. Let $V\hat{a}R(\alpha, T, \hat{\boldsymbol{\rho}}_k)$ be the estimated portfolio's VaR, where $\hat{\boldsymbol{\rho}}_k$ is the $\left(\frac{n!}{2(n-2)!} \times 1\right)$ vector of the estimated pair-wise correlation coefficients $\hat{\rho}_{ij,k}$ corresponding to the kth correlation estimator. We define the **VaR Percentage Error**, $VPE(\alpha, T, \boldsymbol{\rho}, \hat{\boldsymbol{\rho}}_k)$, corresponding to the kth correlation estimator as:

$$VPE(\alpha, T, \boldsymbol{\rho}, \hat{\boldsymbol{\rho}}_k) = \frac{V\hat{a}R(\alpha, T, \hat{\boldsymbol{\rho}}_k) - VaR(\alpha, T, \boldsymbol{\rho})}{VaR(\alpha, T, \boldsymbol{\rho})}. \tag{11}$$

3.1 The Steps

The general setup described by equations (10) and (11) forms the basis to develop the semi-parametric and parametric approaches described in Sections 4 and 5, respectively. These two approaches differ only on the way the correlation estimation error is obtained. Once we obtain the K correlation errors, the two approaches are developed in the following five steps:

Step 1: For each pair of assets, we fix the values of the 'true' correlation coefficients ρ_{ij}.

Step 2: We calculate the 'true' VaR(α, T, ρ) using the 'true' correlation coefficients ρ_{ij}.

Step 3: We calculate the K estimated $V\hat{a}R(\alpha,T,\hat{\rho}_k)$ for the K different correlation estimators, respectively.

Step 4: We calculate $VPE(\alpha,T,\rho,\hat{\rho}_k)$ for each one of the K correlation estimators by using equation (11).

Step 5: We measure the relationship between $VPE(\alpha,T,\rho,\hat{\rho}_k)$ and $\varepsilon_{ij,k}$. This is done by regressing the (Kx1) vector $VPE(\alpha,T,\rho,\hat{\rho}_k)$ on the (Kx1) vector $\varepsilon_{ij,k}$, i.e.:

$$VPE(\alpha,T,\boldsymbol{\rho},\hat{\boldsymbol{\rho}}_{\mathrm{k}})=b*\varepsilon_{ij,k}+u \qquad (12)$$

In both the semi-parametric and parametric approach, the above methodology is applied to linear and non-linear portfolios. This enables us to examine whether the correlation bias effect depends on the type of portfolio. Two methods are used to calculate VaR: variance-covariance and MC simulation. This allows us to study whether the sensitivity of VPE to correlation errors depends on the method used to calculate VaR. We examine this by performing the following two regressions:

$$VPE(\alpha,T,\rho,\hat{\rho}_k)_{VC}=b_1(\varepsilon_{ij,k})+u_1$$

$$VPE(\alpha,T,\boldsymbol{\rho},\hat{\boldsymbol{\rho}}_{\mathrm{k}})_{MC}=b_2(\varepsilon_{ij,k})+u_2 \qquad (13)$$

where $VPE(\alpha, T, \rho, \hat{\rho}_k)_{VC}$ and $VPE(\alpha, T, \rho, \hat{\rho}_k)_{MC}$ are calculated using the variance-covariance and MC simulation method, respectively. The same correlation errors are used to calculate VPE(·)$_{VC}$ and VPE(·)$_{MC}$. Then, the null hypothesis H_0 and the alternative H_1 hypothesis are stated as follows:

$$H_0: b_1 = b_2,$$

$$H_1: b_1 \neq b_2.$$

Acceptance of the null hypothesis implies that the correlation mis-estimation effect on VaR does not depend on the method employed to calculate VaR. In order to test the null hypothesis, we estimate the following regression:

$$VPE^* = b_1\,(\varepsilon^*_{ij,k}) + \delta\,[D * (\varepsilon^*_{ij,k})] + u\,, \tag{14}$$

where VPE* and $\varepsilon^*_{ij,k}$ are ($2K$x1) vectors containing the pooled data from the variables VPE(·)$_{VC}$ and VPE(·)$_{MC}$, and $\varepsilon_{ij,k}$, respectively, used in equation (13). D is a multiplicative dummy variable that is equal to zero for observations from the first set (variance-covariance method) and equal to one for observations from the second set (MC simulation method). Thus, $\delta = b_2 - b_1$.

The equivalent to H_0 and H_1 corresponding H_0^* and H_1^* hypotheses are formulated now as:

$$H_0^*: \delta = 0,$$

$$H_1^*: \delta \neq 0.$$

In the case that the results from the two methods do not differ significantly, δ will be zero. In the case that the H_1^* hypothesis is accepted, the implication is that the sensitivity of VaR mis-calculation to correlation errors depends on the method that is employed to calculate VaR. Hence, if $\delta > (<) 0$, the correlation bias effect is greater (lower) if the MC simulation (variance-covariance) method is used[2].

In Sections 4 and 5, we apply the described methodology within a semi-parametric and a parametric framework, respectively. For the purposes of our study the one-day 95% VaR is calculated.

4 The Semi-Parametric Approach

4.1 The Methodology

In this section, we describe the semi-parametric methodology used to investigate the effect of mis-estimating correlation on VaR. Our approach is semi-parametric in that we do not use specific correlation estimators. On the contrary, we consider a *general* class of *K* estimators. On the other hand, the cost of this is that we must make explicit hypotheses on the correlation error distribution [see also Jacquier and Jarrow (2000) for a discussion on error distributions in the context of option model risk]. We assume that the correlation estimation errors $\varepsilon_{ij,k}$ are random numbers drawn from a normal distribution with mean μ_ε and standard deviation σ_ε, i.e.

$$\varepsilon_{ij,k} \sim N(\mu_\varepsilon, \sigma_\varepsilon), \quad k = 1,\ldots,K. \tag{15}$$

[2] The way we set up the equality coefficients test, allows us to tell which one of the two beta coefficients is greater. In contrast, standard techniques for testing the equality of the regression coefficients such as the Chow or Wald type tests are not informative about the relative size of the coefficients under scrutiny.

We choose $\mu_\varepsilon = 0$ and $\sigma_\varepsilon = 0.03$. Assuming a zero mean implies that we examine a class of unbiased correlation estimators. This choice of σ_ε is based on the empirical evidence on the correlation errors across various estimators, as documented in the papers of Walter and Lopez (2000a, Exhibit 5, p. 75), and Engle (2002, Table 1, p. 345). We calculate the standard deviation of the various measures of correlation errors that are reported in both papers.[3]

Once we specify the correlation errors distribution, the five steps described in Section 3 are implemented. In Sections 4.2 and 4.3 we apply the described semi-parametric approach to linear and non-linear portfolios, respectively.

4.2 Linear Portfolios

We consider the case of a portfolio consisting of two long stocks. We assume that the portfolio position on each asset is 15,000 dollars, and that the standard deviations of the daily returns of each asset are 0.02 and 0.04, respectively. As a first step, we draw 50 random numbers (correlation errors, i.e. $K = 50$) from a normal distribution N(0, 0.03). We calculate 50 portfolio's VPE by considering the 'true' correlation coefficient and each one of the 50 different correlation estimators. Then, the regression described by equation (12) is performed. The analysis is repeated for different 'true' correlation coefficients (ρ = -0.9, -0.8,..., 0.8, 0.9) so as to examine the "mis-estimating correlation" effect on VaR for a range of 'true' correlations. The same 50 correlation errors are used for every

[3] Walter and Lopez (2000a) calculate the one-month mean forecast correlation error for each one of six different cross-currency pairs by using eight different correlation forecast methods. For each cross-currency pair, we calculate the standard deviation of the mean forecast error across the eight different estimators. The standard deviation of the mean forecast error ranges from 0.007 to 0.074 across the cross-currency pairs; it is 0.03 on average. Engle (2002) calculates the mean absolute error using eight different correlation models for various correlation structures. Similarly, for each correlation structure, we calculate the standard deviation across the eight different correlation models. We find that the standard deviation of the correlation error ranges from 0.013 to 0.058 having an average value of 0.03.

‘true’ correlation coefficient. This ensures that the results from our analysis are comparable across the ‘true’ correlations since we do not allow for any variation in the random numbers.

We apply both the DN and the MC simulation methods to calculate VaR. μ_π [equation (3)] and μ_i [i = 1,2, equation (9)] are set equal to zero since the used VaR holding period is short. In the MC simulation application, the initial prices of asset 1 and 2 are set to 100 and 150, respectively. Moreover, we simulate the asset price path by using the same set of random numbers for every ‘true’ correlation value. This allows us to study the VPE and ε_{ij} relationship across ‘true’ correlations, without introducing any ‘noise’ from using different random number streams in the MC simulations.

Table 1 shows the results (beta coefficient and *t*-statistics in parenthesis) from regressing VPE on ε_{ij} [equation (12)] across ‘true’ correlations ranging from –0.9 to 0.9.

Table 1. Correlation Mis-Estimation Coefficient: Semi-Parametric Approach and Linear Portfolios

True Correlation	DN VaR		MC VaR	
0.9	0.233	(1333.2)	0.283	(25.9)
0.8	0.244	(1271.2)	0.207	(34.2)
0.7	0.257	(1209.2)	0.285	(101.7)
0.6	0.270	(1147.1)	0.274	(78.9)
0.5	0.286	(1085.1)	0.297	(30.3)
0.4	0.303	(1023.0)	0.321	(66.5)
0.3	0.323	(961.0)	0.405	(82.9)
0.2	0.345	(898.9)	0.351	(103.6)
0.1	0.371	(836.9)	0.347	(49.5)
-0.1	0.435	(712.8)	0.414	(186.7)
-0.2	0.477	(650.7)	0.443	(66.2)
-0.3	0.527	(588.6)	0.553	(78.4)
-0.4	0.589	(526.5)	0.591	(157.0)
-0.5	0.668	(464.4)	0.694	(191.7)
-0.6	0.770	(402.3)	0.832	(215.9)
-0.7	0.911	(340.1)	0.883	(103.9)
-0.8	1.114	(277.9)	1.196	(173.5)
-0.9	1.434	(215.6)	1.644	(239.2)

Note: Results from regressing VPE on correlation error across various ‘true’ correlation coefficients. The *t*-statistics are reported in parenthesis.

In all cases, the value of the t-statistic indicates that the positive coefficient of the correlation error is significant. The presence of a strong linear relationship between VaR mis-estimation and correlation errors is also confirmed by the high values (more than 95%) of R^2 (not reported here). Thus, the higher the error in estimating correlation, the higher the VaR mis-estimation is. In addition, the coefficient increases as the level of the 'true' correlation decreases for most 'true' correlations. Some exceptions occur for the calculated via MC simulation VaR. This result has an important implication. The VaR error is more sensitive to correlation estimation errors in the case of well-diversified linear portfolios (i.e. portfolios with negatively correlated assets).[4]

Finally, we test whether the sensitivity of VPE to correlation errors depends on the method used to calculate VaR. Table 2 shows the results from the regression described in equation (14). We can see that δ is insignificant only in a few cases (22%). This indicates that in general, the degree to which the correlation mis-estimation affects VaR depends on the method used to calculate VaR. In particular, it seems that the VaR error is more sensitive to correlation mis-estimation when MC simulation is used. This is because δ is significantly positive in most of the remaining cases (71%). Hence, risk managers should place more emphasis on estimating correlation accurately, if MC simulation is used to calculate VaR.

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[4] For the case of DN VaR, it is possible to derive analytical expressions for the sensitivity of VaR mis-estimation to correlation errors as the 'true' correlation changes. VPE can be written as: $VPE(\alpha,T,\rho,\hat{\rho}_k)=\sqrt{1+\frac{2\sum_{i=1}^{n}\sum_{j\neq i}^{n}W_iW_j\sigma_i\sigma_j\varepsilon_{ij,k}}{\sigma_\Pi^2}}-1$. Then, $\frac{\partial\left(\frac{\partial VPE(\alpha,T,\rho,\hat{\rho}_k)}{\partial\varepsilon_{ij,k}}\right)}{\partial\rho_{ij}}=-W_i^2W_j^2\sigma_i^2\sigma_j^2\left(\sigma_\Pi^2\hat{\sigma}_\Pi^2\right)^{-\frac{3}{2}}\left(\sigma_\Pi^2+\hat{\sigma}_\Pi^2\right)<0$.
The negative sign is in line with our results.

Table 2. Comparison of DN and MC Simulation VPE Sensitivity to Correlation Estimation Errors: Semi-Parametric Approach and Linear Portfolios

True Correlation	Estimated δ coefficient
0.9	0.051
0.8	-0.037
0.7	0.028
0.6	0.004*
0.5	0.011*
0.4	0.018
0.3	0.083
0.2	0.006*
0.1	-0.024
-0.1	-0.021
-0.2	-0.033
-0.3	0.026*
-0.4	0.002
-0.5	0.026
-0.6	0.061
-0.7	0.103
-0.8	0.079
-0.9	0.200

Note: Results of the regression $VPE^{*} = b_1 (\varepsilon^{*}_{ij,k}) + \delta [D * (\varepsilon^{*}_{ij,k})] + u$. The asterisks denote *insignificance* at the 5% level.

4.3 Option Portfolios

In this section, we apply the semi-parametric approach to portfolios consisting of two plain-vanilla European call options[5]. The options are written on two non-dividend paying stocks A and B, respectively, that exhibit correlation ρ_{AB}. In total, eighteen different option portfolios are considered; these are constructed for three different times-to-maturity, τ = 10, 30 and 60 days, and six different current levels of moneyness, M = -20%, -10%, -5%, 5%, 10%, 20%[6]. To calculate VaR, an option pricing model needs to be employed. We choose the widely used Black-Scholes (1973) model[7]. The

[5] We also applied the semi-parametric analysis to put option portfolios. The results were similar to the call option portfolios. Therefore, we prefer not to report them. We also found that the results are not affected by the VaR confidence level.

[6] The call "moneyness", $M = (P_i - X_i)/P_i$, is defined as the percentage difference of the ith underlying asset price from the ith option's strike price.

[7] Lehar, Scheicher, and Schittekopf (2002) compare the Black-Scholes, GARCH and stochastic volatility option pricing models in terms of pricing errors and VaR forecasting performance. They find that the more complex models improve on the Black-Scholes methodology only for pricing purposes.

risk-free rate is assumed to be 10%, the daily standard deviations of assets A and B are 0.02 and 0.04, respectively, and the initial stock price of assets A and B are 100 and 150, respectively. VaR is calculated using both the DG approach and the MC simulation approach. The same set of correlation errors is used as in Section 4.2, so as to ensure that the results are comparable between the linear and the non-linear portfolio cases.

Tables 3, 4, and 5 show the estimated beta coefficients from regressing the options portfolio VPE on correlation errors for τ = 10, 30, and 60 days, respectively. The results are reported across various 'true' correlations and moneyness levels. We can see that there is a significant positive relationship between correlation errors and VPE[8]. This holds for both the DG and MC simulation methods.

Table 3. Correlation Mis-Estimation Coefficient: Semi-parametric approach and non-linear portfolios (Options time-to-maturity is 10 days)

True	M= -20%		M= -10%		M= -5%		M= 5%		M= 10%		M= 20%	
Correlation	DG	MC	DG	MC	DG	MC	DG	MC	DG	MC	DG	MC
0.9	0.012	0.002*	0.100	0.048	0.169	0.124	0.223	0.244	0.217	0.258	0.200	0.267
0.8	0.012	0.001*	0.102	0.077	0.174	0.135	0.233	0.179	0.227	0.185	0.209	0.185
0.7	0.012	0.030	0.104	0.084	0.180	0.139	0.244	0.237	0.237	0.249	0.218	0.234
0.6	0.012	0.016	0.105	0.090	0.186	0.155	0.256	0.255	0.249	0.248	0.227	0.221
0.5	0.012	0.010	0.107	0.092	0.193	0.167	0.269	0.253	0.261	0.278	0.238	0.258
0.4	0.012	-0.005	0.109	0.056	0.200	0.164	0.284	0.277	0.276	0.278	0.250	0.327
0.3	0.012	-0.008	0.111	0.092	0.208	0.133	0.301	0.310	0.291	0.341	0.263	0.247
0.2	0.012	0.015	0.113	0.090	0.216	0.173	0.319	0.313	0.309	0.297	0.278	0.286
0.1	0.012	0.012	0.116	0.086	0.224	0.195	0.340	0.283	0.329	0.299	0.294	0.263
-0.1	0.012	0.024	0.120	0.137	0.244	0.236	0.392	0.355	0.377	0.366	0.333	0.334
-0.2	0.012	0.025	0.123	0.148	0.256	0.232	0.424	0.392	0.407	0.391	0.356	0.336
-0.3	0.012	0.024	0.125	0.153	0.268	0.274	0.461	0.435	0.443	0.426	0.383	0.416
-0.4	0.012	0.020	0.128	0.180	0.281	0.334	0.506	0.503	0.485	0.505	0.415	0.441
-0.5	0.012	0.022	0.130	0.181	0.296	0.343	0.560	0.603	0.535	0.573	0.452	0.499
-0.6	0.012	0.041	0.133	0.243	0.313	0.396	0.627	0.690	0.597	0.679	0.497	0.464
-0.7	0.012	0.048	0.136	0.222	0.331	0.536	0.711	0.785	0.676	0.705	0.551	0.586
-0.8	0.012	0.015	0.139	0.319	0.352	0.575	0.822	0.991	0.778	0.886	0.618	0.731
-0.9	0.012	0.017	0.142	0.364	0.375	0.861	0.973	1.440	0.916	1.147	0.705	0.741

Notes: Results from regressing VPE on correlation error for various 'true' correlation coefficients and various levels of moneyness. The time-to-maturity of both options is 10 days. The asterisks denote *insignificance* at the 5% level.

[8] Only four exceptions occur for deep OTM options (M= -20%) having 10 days to maturity.

Table 4. Correlation Mis-Estimation Coefficient: Semi-parametric approach and non-linear portfolios (Options time-to-maturity is 30 days)

True	*M* = -20%		*M* = -10%		*M* = -5%		*M* = 5%		*M* = 10%		*M* = 20%	
Correlation	DG	MC	DG	MC	DG	MC	DG	MC	DG	MC	DG	MC
0.9	0.083	0.059	0.156	0.145	0.185	0.206	0.215	0.260	0.217	0.264	0.209	0.271
0.8	0.084	0.081	0.160	0.151	0.192	0.177	0.224	0.175	0.226	0.178	0.218	0.182
0.7	0.085	0.104	0.165	0.132	0.199	0.195	0.234	0.237	0.237	0.244	0.228	0.248
0.6	0.087	0.131	0.170	0.166	0.207	0.217	0.245	0.216	0.248	0.229	0.239	0.219
0.5	0.088	0.058	0.175	0.212	0.215	0.211	0.257	0.272	0.261	0.298	0.251	0.284
0.4	0.089	0.089	0.181	0.151	0.224	0.239	0.271	0.279	0.275	0.291	0.264	0.318
0.3	0.090	0.073	0.187	0.146	0.234	0.185	0.286	0.300	0.291	0.318	0.278	0.279
0.2	0.091	0.063	0.194	0.156	0.244	0.217	0.303	0.252	0.308	0.265	0.294	0.261
0.1	0.092	0.078	0.201	0.158	0.256	0.218	0.321	0.298	0.328	0.323	0.313	0.308
-0.1	0.095	0.110	0.216	0.202	0.283	0.277	0.367	0.313	0.376	0.315	0.357	0.324
-0.2	0.096	0.117	0.225	0.239	0.299	0.291	0.395	0.389	0.406	0.400	0.384	0.392
-0.3	0.098	0.148	0.235	0.273	0.316	0.333	0.428	0.416	0.441	0.431	0.415	0.421
-0.4	0.099	0.116	0.245	0.299	0.336	0.371	0.466	0.474	0.483	0.495	0.452	0.475
-0.5	0.100	0.116	0.256	0.278	0.358	0.431	0.512	0.546	0.533	0.566	0.497	0.541
-0.6	0.102	0.136	0.268	0.328	0.383	0.426	0.567	0.645	0.594	0.660	0.551	0.526
-0.7	0.103	0.189	0.281	0.379	0.412	0.504	0.636	0.692	0.671	0.709	0.618	0.641
-0.8	0.105	0.166	0.296	0.452	0.446	0.636	0.724	0.868	0.771	0.880	0.704	0.816
-0.9	0.106	0.153	0.312	0.476	0.485	0.741	0.839	1.148	0.906	1.152	0.817	0.901

Note: Results from regressing VPE on correlation error for various 'true' correlation coefficients and various levels of moneyness. The time-to-maturity of both options is 30 days. The asterisks denote *insignificance* at the 5% level.

Table 5. Correlation Mis-Estimation Coefficient: Semi-parametric approach and non-linear portfolios (Options time-to-maturity is 60 days)

True	*M* = -20%		*M* = -10%		*M* = -5%		*M* = 5%		*M* = 10%		*M* = 20%	
Correlation	DG	MC	DG	MC	DG	MC	DG	MC	DG	MC	DG	MC
0.9	0.129	0.126	0.173	0.198	0.190	0.232	0.210	0.264	0.213	0.267	0.212	0.271
0.8	0.132	0.145	0.179	0.175	0.197	0.184	0.218	0.184	0.222	0.181	0.221	0.182
0.7	0.135	0.132	0.185	0.173	0.205	0.206	0.228	0.235	0.233	0.245	0.231	0.250
0.6	0.138	0.151	0.192	0.220	0.213	0.232	0.239	0.212	0.244	0.216	0.242	0.220
0.5	0.142	0.162	0.199	0.226	0.222	0.221	0.250	0.256	0.256	0.275	0.254	0.287
0.4	0.145	0.096	0.206	0.201	0.231	0.262	0.263	0.304	0.269	0.304	0.268	0.318
0.3	0.149	0.127	0.214	0.169	0.242	0.188	0.277	0.275	0.284	0.291	0.282	0.288
0.2	0.153	0.124	0.223	0.207	0.253	0.247	0.293	0.246	0.301	0.247	0.299	0.257
0.1	0.157	0.160	0.232	0.208	0.266	0.243	0.310	0.301	0.319	0.319	0.318	0.315
-0.1	0.165	0.179	0.254	0.247	0.295	0.292	0.352	0.331	0.365	0.329	0.364	0.334
-0.2	0.170	0.199	0.266	0.284	0.313	0.316	0.378	0.362	0.393	0.386	0.392	0.411
-0.3	0.175	0.225	0.280	0.316	0.332	0.342	0.408	0.412	0.425	0.426	0.424	0.425
-0.4	0.180	0.192	0.295	0.357	0.354	0.382	0.442	0.464	0.464	0.482	0.463	0.482
-0.5	0.185	0.203	0.312	0.324	0.379	0.458	0.483	0.536	0.509	0.546	0.509	0.548
-0.6	0.191	0.249	0.330	0.370	0.407	0.445	0.533	0.548	0.565	0.591	0.566	0.553
-0.7	0.197	0.316	0.351	0.427	0.440	0.533	0.593	0.642	0.634	0.675	0.637	0.662
-0.8	0.203	0.323	0.374	0.552	0.479	0.662	0.668	0.818	0.722	0.856	0.728	0.851
-0.9	0.210	0.286	0.400	0.537	0.524	0.694	0.765	0.991	0.839	1.044	0.848	0.970

Note: Results from regressing VPE on correlation error for various 'true' correlation coefficients and various levels of moneyness. The time-to-maturity of both options is 60 days. The asterisks denote *insignificance* at the 5% level.

The magnitude of the beta coefficient changes as the 'true' correlation, the moneyness level, and the time-to-maturity changes. In most cases, for any given time-to-maturity and moneyness level, the sensitivity of VPE to correlation errors increases as the 'true' correlation decreases. This is consistent with the linear portfolio case results[9]. Some exceptions occur when VaR is calculated with MC simulation, and the 'true' correlation is positive.

For any given 'true' correlation, the sensitivity of VPE to correlation errors across the three expiries and the various levels of moneyness takes its maximum value for $\tau = 10\%$ and $M = 5\%$ and 10%. This result imply that the risk manager should be more concerned about the choice of the correlation estimator in the case that the institution's portfolio contain short-dated options within the range of 5% - 10% moneyness level.

Finally, we test whether the sensitivity of VPE to correlation errors depends on the method used to calculate VaR. Tables 6, 7, and 8 show the results from running the regression described by equation (14) when the time-to-maturity is 10, 30 and 60 days, respectively. In most of the option portfolios (82%), the correlation bias effect on VaR depends on the employed method. In the majority of such significant cases (66%), the effect from the correlation bias is greater when the MC simulation method is used. These results are consistent with the linear portfolio case.

[9] This result does not necessarily imply that the VaR of well-diversified option portfolios is more exposed to correlation errors. This is because the option portfolios may not be diversified, even though the underlying assets portfolio may be well diversified (the sign of correlation though, will be the same for both the option and the underlying asset portfolio). The extend to which the magnitude of the option portfolio correlation will be the same as that of the underlying asset's portfolio depends on the options delta, i.e. it depends on the options moneyness level and the time-to-maturity.

Table 6. Comparison of DG and MC simulation VPE Sensitivity to Correlation Estimation Errors: Semi-parametric Approach and Non-linear Portfolios (Options time-to-maturity is 10 days)

True Correlation	M= –20%	M = -10%	M = -5%	M = - 5%	M = 10%	M = 20%
0.9	0.248	0.160	0.091	0.037	0.043	0.060
0.8	-0.010	-0.030	-0.040	-0.050	-0.040	-0.020
0.7	0.018	-0.020	-0.040	-0.010	0.012	0.016
0.6	0.004	-0.030	0.028*	-0.000	0.028	-0.010*
0.5	-0.000*	-0.030	0.017	-0.020	0.017	0.017
0.4	-0.020	-0.050	-0.040	-0.010	0.003	0.077
0.3	-0.020	-0.020	-0.080	0.009	0.050	-0.020*
0.2	0.003*	-0.020	-0.040	-0.010	-0.010	0.009*
0.1	0.000*	-0.030	-0.030	-0.060	-0.030	-0.030
-0.1	0.012	0.017	-0.010	-0.040	-0.010	0.002*
-0.2	0.014	0.026	-0.020	-0.030	-0.020	-0.020
-0.3	0.012	0.028	-0.020	-0.030	-0.020	0.033
-0.4	0.008	0.052	0.020	-0.000	0.020	0.026
-0.5	0.011	0.051	0.037	0.044	0.037	0.047
-0.6	0.029	0.109	0.082	0.063	0.082	-0.030
-0.7	0.035	0.079	0.024*	0.071	0.024	0.034
-0.8	0.003*	0.168	0.208	0.159	0.100	0.109
-0.9	0.007	0.206	0.451	0.435	0.221	0.034

Notes: Results of the regression $VPE^* = b_1 (\varepsilon^*_{ij,k}) + \delta [D * (\varepsilon^*_{ij,k})] + u$. The time-to-maturity of the options is 10 days. The asterisks denote *insignificance* at the 5% level.

Table 7. Comparison of DG and MC Simulation VPE Sensitivity to Correlation Estimation Errors: Semi-parametric Approach and Non-linear Portfolios (Options time-to-maturity is 30 days)

True Correlation	M= –20%	M = -10%	M = -5%	M = - 5%	M = 10%	M = 20%
0.9	0.177	0.105	0.075	0.046	0.043	0.051
0.8	-0.000*	-0.009	-0.014	-0.049	-0.049	-0.04
0.7	0.018	-0.033	-0.004*	0.003*	0.007	0.020
0.6	0.044	-0.004*	0.010	-0.063	-0.020	-0.02
0.5	-0.030	0.037	-0.004*	0.019	0.037	0.033
0.4	0.000*	-0.030	0.015	-0.012*	0.016	0.054
0.3	-0.020	-0.041	-0.049	0.014	0.027	0.001
0.2	-0.030	-0.038	-0.027	-0.050	-0.043	-0.03
0.1	-0.010	-0.043	-0.038	-0.024	-0.005	-0.00*
-0.1	0.015	-0.014	-0.006	-0.054	-0.061	-0.03
-0.2	0.021	0.014	-0.008	-0.006*	-0.006	0.008*
-0.3	0.038	0.038	0.016	-0.012*	-0.010	0.006*
-0.4	0.055	0.055	0.035	0.008	0.012	0.022
-0.5	0.022	0.022	0.073	0.034	0.033	0.044
-0.6	0.034	0.060	0.043	0.077	0.066	-0.02
-0.7	0.082	0.087	0.086	0.051	0.035	0.020*
-0.8	0.059	0.142	0.178	0.131	0.099	0.104
-0.9	0.040	0.155	0.242	0.293	0.235	0.079

Notes: Results of the regression $VPE^{*} = b_1 (\varepsilon^{*}_{ij,k}) + \delta [D * (\varepsilon^{*}_{ij,k})] + u$. The time-to-maturity of the options is 30 days. The asterisks denote *insignificance* at the 5% level.

Table 8. Comparison of DG and MC Simulation VPE Sensitivity to Correlation Estimation Errors: Semi-parametric Approach and Non-linear Portfolios (Options time-to-maturity is 60 days)

True Correlation	M= –20%	M= -10%	M= -5%	M= - 5%	M= 10%	M= 20%
0.9	0.131	0.087	0.070	0.051	-0.008*	-0.01*
0.8	0.013	-0.004	-0.013*	-0.035	-0.029	-0.03
0.7	-0.003*	-0.012	0.001*	0.007*	0.001*	0.010
0.6	0.012	0.029	0.019	-0.026	-0.002*	0.009
0.5	0.021	0.027	0.000*	-0.002*	0.002*	0.008
0.4	-0.049	-0.005	0.030	-0.017	0.004*	0.012
0.3	-0.022	-0.045	-0.054	-0.002*	0.024	0.030
0.2	-0.029	-0.016	-0.006*	0.003*	0.007	0.012
0.1	0.004*	-0.025	-0.023	-0.010*	0.001*	0.010
-0.1	0.014	-0.007	-0.003*	-0.022	-0.039	-0.03
-0.2	0.029	0.018	0.003*	-0.016	-0.016	0.001*
-0.3	0.050	0.036	0.010	0.005*	-0.010*	-0.00*
-0.4	0.012*	0.062	0.028	0.022	0.021	0.020
-0.5	0.018	0.013	0.079	0.052	0.023	0.025
-0.6	0.058	0.040	0.038	0.016*	0.038	0.023
-0.7	0.110	0.070	0.089	0.046	0.085	0.070
-0.8	0.113	0.169	0.171	0.136	0.079	0.064
-0.9	0.072	0.127	0.158	0.217	0.203	0.165

Notes: Results of the regression $VPE^{*} = b_1 (\varepsilon^{*}_{ij,k}) + \delta [D * (\varepsilon^{*}_{ij,k})] + u$. The time-to-maturity of the options is 60 days. The asterisks denote *insignificance* at the 5% level.

5 The Parametric Approach

So far, we have conducted a semi–parametric analysis. In this section, we perform a parametric analysis by using *specific* correlation estimators. The effect of mis-estimating correlation to the induced VaR error is studied by using three widely–used specific correlation estimators: the moving average (MA), the exponentially weighted moving average (EWMA), and the GARCH BEKK model.

The specific estimators are applied to a simulated time series of the portfolio's asset prices. The simulated sample is generated by joint MC simulation using a known 'true' correlation value and the volatilities assumed in Section 4.2. Then, the $VPE(\alpha, T, \rho, \hat{\rho}_k)$ is calculated for each 'true' correlation, and the specific correlation estimators $\hat{\rho}_{ij,k}$. Hence, in contrast to the semi-parametric approach, the correlation error is not drawn directly from an assumed distribution. It is determined implicitly as the difference between the estimated correlation and the 'true' correlation. Next, we describe the used correlation estimators, and the results from the parametric analysis.

5.1 The Correlation Estimators

The moving average (or rolling) correlation estimator of two time-series is measured over a rolling window of specific length consisting of consecutive data points taken from the two time series. The *h*-day moving average correlation estimator between assets *i* and *j* at time *T* is defined as follows:

$$\hat{\rho}_{ij} = \frac{\sum_{t=T-h}^{T-1} (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j)}{\sqrt{\sum_{t=T-h}^{T-1} (R_{i,t} - \bar{R}_i)^2} \sqrt{\sum_{t=T-h}^{T-1} (R_{j,t} - \bar{R}_j)}}. \tag{16}$$

where $R_{i,t}$ and $R_{j,t}$ are the returns of asset *i* and *j* at time *t*, respectively. $\bar{R}_i$ and $\bar{R}_j$ are the mean returns of assets *i* and *j*, respectively.

The proposed by RiskMetrics[TM] EWMA correlation estimator uses a smoothing constant λ ($0<\lambda<1$), so as to take into account the most recent data. The EWMA correlation estimator at time *T* is defined as:

$$\hat{\rho}_{ij} = \frac{\sum_{t=T-h}^{T-1} \lambda^{T-t-1} (R_{i,t} - R_i)(R_{j,t} - \bar{R}_j)}{\sqrt{\left(\sum_{t=T-h}^{T-1} \lambda^{T-t-1} (R_{i,t} - \bar{R}_i)^2\right)\left(\sum_{t=T-h}^{T-1} \lambda^{T-t-1} (R_{j,t} - \bar{R}_j)^2\right)}}. \tag{17}$$

The employed scalar and diagonal BEKK models are multivariate GARCH(1,1) models that are nested in the full BEKK model [Engle and Kroner (1995)] written as:

$$\mathbf{H}_t = \mathbf{\Omega} + \mathbf{A}'\varepsilon_{t-1}\mathbf{A} + \mathbf{B}'\mathbf{H}_{t-1}\mathbf{B}, \tag{18}$$

where $\boldsymbol{H_t}$ is the $(n$x$n)$ conditional variance-covariance matrix at time t with diagonal elements $h_{i,t}^2$, and off-diagonal elements $h_{ij,t}$. $\boldsymbol{\Omega}$ is a $(n$x$1)$ parameter vector, and $\boldsymbol{A}$, $\boldsymbol{B}$ are $(n$x$n)$ parameter matrices. In the case of scalar BEKK $\boldsymbol{A}$ and $\boldsymbol{B}$ are scalar parameters and, in the case of diagonal BEKK, $\boldsymbol{A}$ and $\boldsymbol{B}$ are diagonal $(n$x$n)$ matrices. The correlation estimates are derived as follows:

$$\hat{\rho}_{ij,t} = \frac{\hat{h}_{ij,t}}{\hat{h}_{i,t}\hat{h}_{j,t}}, \tag{19}$$

where $\hat{h}_{i,t}$ and $\hat{h}_{j,t}$ are the estimated conditional variances of assets i and j, respectively, and $\hat{h}_{ij,t}$ is the estimated conditional covariance between assets i and j.

Thirteen different models are used to estimate correlations: six MA models corresponding to six different rolling windows (h = 20, 30, 60, 90, 120), five EWMA models corresponding to five different lambda parameters (λ = 0.7, 0.8, 0.9, 0.94, 0.97, 0.99), a scalar BEKK GARCH (1,1) model, and a diagonal BEKK GARCH(1,1) model. Hence, the VPE_{VC} and VPE_{MC} series used in equations (13) and (14) are (13x1) vectors, respectively. The portfolio profiles, as well as the MC simulation inputs are the same with those used in the semi-parametric analysis.

5.2 Linear Portfolios

We apply the parametric approach to the linear two-assets portfolio. Table 9 shows the results from regressing VPE on correlation error [equation (12)] for various 'true' correlation coefficients. The beta coefficient is positive and significant in all cases. In addition, the sensitivity of VPE to correlation errors decreases as the 'true' correlation increases. Some exceptions occur for positively correlated assets in the case that VaR is calculated via MC simulation.

Table 9. Correlation Mis-Estimation Coefficient: Parametric Approach and Linear Portfolios

True Correlation	DN VaR		MC VaR	
0.9	0.233	(341.4)	0.231	(17.3)
0.8	0.243	(509.6)	0.223	(19.7)
0.7	0.251	(282.8)	0.259	(56.9)
0.6	0.262	(274.2)	0.281	(94.2)
0.5	0.275	(250.1)	0.270	(79.1)
0.4	0.289	(212.3)	0.308	(69.9)
0.3	0.306	(193.9)	0.336	(70.8)
0.2	0.326	(175.1)	0.358	(91.0)
0.1	0.350	(162.5)	0.360	(95.4)
-0.1	0.420	(224.1)	0.404	162.0)
-0.2	0.465	(210.7)	0.424	(55.4)
-0.3	0.515	(145.2)	0.516	(62.9)
-0.4	0.593	(83.8)	0.606	(53.2)
-0.5	0.630	(200.3)	0.650	(118.9)
-0.6	0.814	(61.7)	0.846	(76.8)
-0.7	0.968	(63.1)	1.015	(45.7)
-0.8	1.066	(193.9)	1.114	(85.6)
-0.9	1.328	(115.7)	1.520	(61.1)

Note: Results from regressing VPE on correlation error across various 'true' correlation coefficients. The *t*-statistics are reported in parenthesis.

Table 10 shows the results from running the regression described by equation (14). The results regarding the δ coefficient significance are somewhat mixed. In 50% of the cases the δ coefficient is not significant. However, in 78% of the significant cases the δ coefficient is positive. These results indicate that the use of the MC simulation to calculate VaR magnifies any correlation bias effect on VaR, just as it was the case with the semi-parametric approach.

Table 10. Comparison of DN and MC Simulation VPE Sensitivity to Correlation Estimation Errors: Parametric Approach and Linear Portfolios

True Correlation	Estimated δ coefficient
0.9	-0.002*
0.8	-0.020*
0.7	0.008*
0.6	0.019
0.5	-0.005*
0.4	0.031
0.3	0.031
0.2	0.032
0.1	0.009
-0.1	-0.015
-0.2	-0.041
-0.3	0.000*
-0.4	0.013*
-0.5	0.020
-0.6	0.032*
-0.7	0.048*
-0.8	0.049
-0.9	0.922*

Note: Results of the regression $VPE^{*} = b_1\,(\varepsilon^{*}_{ij,k}) + \delta\,[D * (\varepsilon^{*}_{ij,k})] + u$. The asterisks denote *insignificance* at the 5% level.

5.3 Option Portfolios

In this section, we apply the parametric approach to portfolios containing options. Tables 11, 12, and 13 show the correlation mis-estimation coefficient b from regressing VPE to correlation errors for 10, 30 and 60 days to maturity, respectively. The results are reported for different 'true' correlation values and across various levels of moneyness (–20%, -10%, -5%, 5%, 10%, and 20%).

Table 11. Correlation Mis-Estimation Coefficient: Parametric Approach and Non-linear Portfolios (Options time-to-maturity is 10 days)

True	*M* = -20%		*M* = -10%		*M* = -5%		*M* = 5%		*M* = 10%		*M* = 20%	
Correlation	DG	MC	DG	MC	DG	MC	DG	MC	DG	MC	DG	MC
0.9	0.0117	0.006*	0.100	0.063	0.169	0.122	0.223	0.201	0.217	0.207	0.200	0.216
0.8	0.0123	0.005*	0.102	0.076	0.174	0.109	0.232	0.188	0.226	0.197	0.208	0.194
0.7	0.0120	0.017	0.103	0.067	0.178	0.137	0.239	0.232	0.233	0.239	0.214	0.233
0.6	0.0119	0.023	0.104	0.076	0.183	0.132	0.249	0.243	0.242	0.250	0.222	0.227
0.5	0.0121	0.013	0.106	0.086	0.189	0.149	0.260	0.239	0.252	0.253	0.231	0.244
0.4	0.0122	0.012	0.108	0.079	0.195	0.159	0.273	0.264	0.264	0.271	0.241	0.271
0.3	0.0123	0.006	0.110	0.078	0.201	-0.004	0.287	0.281	0.278	0.293	0.252	0.283
0.2	0.0118	0.005	0.112	0.083	0.209	0.156	0.304	0.302	0.294	0.316	0.265	0.274
0.1	0.0119	0.006	0.114	0.085	0.218	0.170	0.324	0.305	0.313	0.309	0.281	0.292
-0.1	0.0122	0.020	0.120	0.109	0.240	0.203	0.380	0.351	0.366	0.353	0.324	0.289
-0.2	0.0116	0.027	0.122	0.139	0.253	0.240	0.415	0.382	0.400	0.367	0.350	0.359
-0.3	0.0121	0.018	0.124	0.149	0.265	0.251	0.453	0.418	0.435	0.429	0.377	0.378
-0.4	0.0122	0.020	0.128	0.171	0.282	0.320	0.508	0.508	0.487	0.502	0.417	0.447
-0.5	0.0120	0.024	0.130	0.170	0.290	0.336	0.535	0.539	0.512	0.522	0.435	0.466
-0.6	0.0116	0.032	0.134	0.237	0.317	0.455	0.651	0.727	0.620	0.663	0.513	0.533
-0.7	0.0119	0.027	0.137	0.278	0.336	0.547	0.740	0.868	0.703	0.781	0.569	0.623
-0.8	0.0121	0.022	0.139	0.290	0.348	0.571	0.798	0.931	0.755	0.819	0.603	0.695
-0.9	0.0115	0.019	0.142	0.343	0.368	0.693	0.926	1.238	0.871	1.096	0.676	0.768

Notes: Results from regressing VPE on correlation error for various 'true' correlation coefficients and various levels of moneyness. The time-to-maturity of both options is 10 days. The asterisks denote *insignificance* at the 5% level.

Table 12. Correlation Mis-Estimation Coefficient: Parametric Approach and Non-linear Portfolios (Options time-to-maturity is 30 days)

True	*M* = -20%		*M* = -10%		*M* = -5%		*M* = 5%		*M* = 10%		*M* = 20%	
Correlation	DG	MC	DG	MC	DG	MC	DG	MC	DG	MC	DG	MC
0.9	0.083	0.082	0.156	0.137	0.185	0.166	0.215	0.201	0.217	0.205	0.209	0.205
0.8	0.084	0.082	0.160	0.134	0.191	0.160	0.223	0.188	0.226	0.188	0.218	0.192
0.7	0.085	0.081	0.163	0.155	0.196	0.192	0.230	0.226	0.232	0.238	0.224	0.232
0.6	0.086	0.090	0.167	0.153	0.202	0.187	0.239	0.240	0.242	0.242	0.232	0.238
0.5	0.087	0.098	0.172	0.166	0.209	0.200	0.249	0.240	0.252	0.255	0.242	0.254
0.4	0.088	0.080	0.177	0.183	0.217	0.221	0.260	0.258	0.264	0.271	0.253	0.268
0.3	0.089	0.078	0.182	0.167	0.226	0.220	0.273	0.288	0.278	0.302	0.266	0.293
0.2	0.091	0.066	0.189	0.175	0.236	0.211	0.288	0.298	0.293	0.308	0.280	0.293
0.1	0.092	0.070	0.196	0.174	0.247	0.215	0.307	0.287	0.313	0.306	0.298	0.314
-0.1	0.095	0.099	0.213	0.200	0.277	0.244	0.357	0.335	0.365	0.340	0.347	0.310
-0.2	0.096	0.108	0.223	0.232	0.295	0.281	0.388	0.362	0.398	0.376	0.377	0.374
-0.3	0.097	0.074	0.233	0.262	0.313	0.320	0.420	0.402	0.433	0.426	0.408	0.420
-0.4	0.099	0.118	0.245	0.286	0.337	0.380	0.468	0.488	0.485	0.505	0.454	0.478
-0.5	0.100	0.109	0.251	0.304	0.348	0.394	0.491	0.519	0.510	0.531	0.476	0.503
-0.6	0.102	0.166	0.271	0.356	0.391	0.477	0.587	0.646	0.616	0.662	0.571	0.593
-0.7	0.104	0.172	0.285	0.424	0.420	0.559	0.659	0.771	0.698	0.789	0.641	0.703
-0.8	0.105	0.202	0.293	0.425	0.439	0.599	0.705	0.791	0.749	0.814	0.684	0.768
-0.9	0.106	0.141	0.307	0.485	0.473	0.722	0.803	1.058	0.863	1.079	0.780	0.887

Notes: Results from regressing VPE on correlation error for various 'true' correlation coefficients and various levels of moneyness. The time-to-maturity of both options is 30 days. The asterisks denote *insignificance* at the 5% level.

Table 13. Correlation Mis-Estimation Coefficient: Parametric Approach and Non-linear Portfolios (Options time-to-maturity is 60 days)

True	M= -20%		M= -10%		M= -5%		M= 5%		M= 10%		M= 20%	
Correlation	DG	MC	DG	MC	DG	MC	DG	MC	DG	MC	DG	MC
0.9	0.129	0.138	0.173	0.165	0.190	0.187	0.210	0.213	0.213	0.206	0.212	0.205
0.8	0.132	0.136	0.179	0.159	0.197	0.172	0.218	0.191	0.222	0.193	0.220	0.191
0.7	0.134	0.125	0.183	0.187	0.201	0.203	0.224	0.228	0.228	0.229	0.227	0.237
0.6	0.137	0.137	0.188	0.191	0.208	0.197	0.233	0.231	0.237	0.235	0.235	0.244
0.5	0.140	0.158	0.194	0.194	0.216	0.212	0.242	0.240	0.247	0.249	0.245	0.253
0.4	0.143	0.146	0.201	0.217	0.224	0.243	0.253	0.262	0.259	0.263	0.257	0.269
0.3	0.146	0.143	0.208	0.203	0.233	0.239	0.265	0.285	0.271	0.295	0.270	0.299
0.2	0.150	0.141	0.216	0.208	0.244	0.229	0.279	0.278	0.287	0.294	0.285	0.297
0.1	0.154	0.132	0.226	0.214	0.256	0.240	0.297	0.296	0.305	0.306	0.303	0.314
-0.1	0.164	0.154	0.250	0.239	0.289	0.263	0.343	0.309	0.355	0.315	0.353	0.324
-0.2	0.169	0.188	0.264	0.270	0.308	0.304	0.371	0.348	0.386	0.370	0.384	0.385
-0.3	0.174	0.212	0.277	0.294	0.328	0.343	0.401	0.403	0.418	0.409	0.417	0.416
-0.4	0.180	0.203	0.296	0.338	0.355	0.396	0.444	0.471	0.465	0.487	0.465	0.485
-0.5	0.184	0.217	0.305	0.371	0.368	0.421	0.465	0.493	0.488	0.511	0.488	0.513
-0.6	0.192	0.267	0.335	0.402	0.416	0.486	0.550	0.593	0.585	0.623	0.587	0.610
-0.7	0.198	0.294	0.356	0.476	0.450	0.571	0.613	0.709	0.658	0.743	0.661	0.732
-0.8	0.202	0.320	0.370	0.526	0.471	0.605	0.652	0.736	0.703	0.782	0.707	0.771
-0.9	0.208	0.318	0.393	0.558	0.511	0.698	0.734	0.951	0.802	1.004	0.809	0.974

Notes: Results from regressing VPE on correlation error for various 'true' correlation coefficients and various levels of moneyness. The time-to-maturity of both options is 60 days. The asterisks denote *insignificance* at the 5% level.

We can see that for both the DG and MC simulation methods there is a strong linear positive relationship between VPE and correlation errors[10]. These results are in line with those obtained from the semi-parametric analysis on the options portfolios. The magnitude of the correlation error coefficient depends on the 'true' correlation, the moneyness level and the time-to-maturity in the same way as in the semi-parametric approach.

In most of the cases, for a given level of moneyness and time-to-maturity, the sensitivity of VPE to correlation errors decreases as the 'true' correlation increases. (This does not hold for a number of cases when VaR is calculated via MC simulation and the 'true' correlation is positive). Moreover, for any given level of 'true' correlation, the correlation error coefficient takes its maximum value in the case of short-dated ITM options (M=5% and 10%).

[10] Two exceptions occur for deep OTM option portfolios (M= -20%) and 10 days to maturity.

Tables 14, 15, and 16 show the results from testing whether the effect of correlation errors on VaR depends on the method used to calculate VaR; the results are reported for 10, 30 and 60 days-to-maturity, respectively. We can see that in 76% of the cases the correlation bias effect depends on the method used to calculate VaR. In 65% of these cases, the beta coefficient is higher when VaR is calculated using MC simulation. This implies that the option portfolios calculated VaR is more sensitive to correlation errors, in the case that MC simulation is used to calculate VaR.

Table 14. Comparison of DG and MC Simulation VPE Sensitivity to Correlation Estimation Errors: Parametric Approach and Non-linear Portfolios (Options time-to-maturity is 10 days)

True Correlation	*M*= –20%	*M* = -10%	*M* = -5%	*M* = - 5%	*M* = 10%	*M* = 20%
0.9	-0.006*	-0.037	-0.047	-0.022	-0.010*	0.016*
0.8	-0.007*	-0.026	-0.065*	-0.045	-0.029	-0.014*
0.7	0.005	-0.036	-0.041	-0.007	0.007	0.019
0.6	0.011	-0.029	-0.051	-0.006*	0.008	0.006*
0.5	0.001*	-0.020	-0.039	-0.021	0.001*	0.013
0.4	-0.001*	-0.029	-0.036	-0.009	0.006*	0.031
0.3	0.000	-0.032	-0.206	-0.006	0.016	0.031
0.2	-0.007	-0.029	-0.053	-0.001*	0.022	0.009
0.1	-0.005	-0.030	-0.048	-0.019	-0.004*	0.011
-0.1	0.008	-0.011	-0.037	-0.029	-0.013	-0.035
-0.2	0.015	0.017	-0.013	-0.034	-0.032	0.009*
-0.3	0.006*	0.025	-0.014*	-0.035	-0.006*	0.000*
-0.4	0.007	0.043	0.038	0.000*	0.015*	0.030
-0.5	0.012	0.040	0.046	0.004*	0.010*	0.032
-0.6	0.020	0.103	0.138	0.076	0.043	0.020*
-0.7	0.015	0.141	0.211	0.128	0.077	0.054
-0.8	0.010	0.151	0.224	0.133	0.064	0.092
-0.9	0.008	0.202	0.325	0.312	0.225	0.091

Notes: Results of the regression $VPE^* = b_1\,(\varepsilon^*_{ij,k}) + \delta\,[D * (\varepsilon^*_{ij,k})] + u$. The time-to-maturity of the options is 10 days. The asterisks denote *insignificance* at the 5% level.

Table 15. Comparison of DG and MC Simulation VPE Sensitivity to Correlation Estimation Errors: Parametric Approach and Non-linear Portfolios (Options time-to-maturity is 30 days)

True Correlation	*M*= –20%	*M* = -10%	*M* = -5%	*M* = - 5%	*M* = 10%	*M* = 20%
0.9	-0.002*	-0.018	-0.019*	-0.014*	-0.011*	-0.004*
0.8	-0.002*	-0.026	-0.031	-0.035	-0.038	-0.026
0.7	-0.004*	-0.008	-0.003*	-0.003*	0.005	0.009
0.6	0.004	-0.014	-0.016	0.002*	0.000*	0.006
0.5	0.011	-0.006*	-0.009	-0.009	0.003*	0.003*
0.4	-0.008	0.007	0.004*	-0.003*	0.007*	0.015
0.3	-0.012	-0.016	-0.005	0.015	0.025	0.028
0.2	-0.024	-0.013	0.015	0.009	0.015	0.012
0.1	-0.022	-0.022	-0.032	-0.020	-0.007*	-0.007*
-0.1	0.004	-0.013	-0.033	-0.022	-0.026	-0.037
-0.2	0.012	0.009	-0.014	-0.026	-0.023	-0.003*
-0.3	-0.023*	0.029	0.008*	-0.018*	-0.008*	0.012
-0.4	0.019	0.041	0.043	0.020	0.020	0.023
-0.5	0.009	0.052	0.046	0.028	0.021	0.027
-0.6	0.064	0.085	0.086	0.059	0.045	0.022
-0.7	0.068	0.140	0.139	0.112	0.091	0.061
-0.8	0.097	0.132	0.160	0.086	0.065	0.084
-0.9	0.035	0.178	0.249	0.255	0.217	0.107

Notes: Results of the regression $VPE^* = b_1\ (\varepsilon^*_{ij,k}) + \delta\ [D * (\varepsilon^*_{ij,k})] + u$. The time-to-maturity of the options is 30 days. The asterisks denote *insignificance* at the 5% level.

Table 16. Comparison of DG and MC Simulation VPE Sensitivity to Correlation Estimation Errors: Parametric Approach and Non-linear Portfolios (Options time-to-maturity is 60 days)

True Correlation	*M*= –20%	*M* = -10%	*M* = -5%	*M* = - 5%	*M* = 10%	*M* = 20%
0.9	0.008*	-0.008*	-0.003*	0.004*	-0.006*	0.005*
0.8	0.004*	-0.020	-0.025*	-0.027	-0.025	-0.026*
0.7	-0.009	0.005*	0.002*	0.004	0.011	0.006
0.6	0.000*	0.003*	-0.011	0.000	0.183	-0.008
0.5	0.018	0.001*	-0.004*	-0.003*	0.006*	-0.002*
0.4	0.003*	0.016	0.019	0.009	0.012	0.014
0.3	-0.003*	-0.005*	0.006	0.020	0.022	0.013
0.2	-0.008	-0.008	-0.014	-0.001*	0.030	-0.011
0.1	-0.022	-0.011	-0.016	-0.001*	0.004*	-0.011
-0.1	-0.010	-0.011	-0.026	-0.034	-0.002*	-0.064
-0.2	0.019	0.006*	-0.004*	-0.023	-0.018	-0.025
-0.3	0.038	0.016	0.016	0.002*	0.010*	-0.039
-0.4	0.023	0.042	0.041	0.027	0.036	-0.018
-0.5	0.034	0.066	0.053	0.028	0.034	-0.021
-0.6	0.075	0.066	0.070	0.044	0.078	-0.054
-0.7	0.096	0.119	0.121	0.096	0.123	-0.038
-0.8	0.118	0.156	0.134	0.084	0.116	-0.012*
-0.9	0.109	0.166	0.187	0.216	0.295	-0.042

Notes: Results of the regression $VPE^* = b_1\ (\varepsilon^*_{ij,k}) + \delta\ [D * (\varepsilon^*_{ij,k})] + u$. The time-to-maturity of the options is 60 days. The asterisks denote *insignificance* at the 5% level.

6. Conclusions

We have examined explicitly the systematic relationship between errors in estimating correlation and the resulting error in calculating VaR (Percentage VaR Error, VPE). Understanding this relationship is important for measuring market risk accurately. First, a semi-parametric approach that considers a general class of unbiased correlation estimators was developed; correlation estimation errors were simulated from an assumed normal distribution. Then, the effect of mis-estimating correlation on VaR was studied parametrically by applying specific correlation estimators (moving average, exponentially weighted moving average and GARCH-type models) to simulated data. Both approaches were implemented using specific linear and non-linear portfolios containing options. VaR was calculated with variance-covariance (delta-normal and delta-gamma), and Monte Carlo (MC) simulation methods. Using various portfolios and techniques to calculate VaR enabled us to explore whether the sensitivity of the mis-calculated VaR to correlation errors depends on the type of portfolio and the employed method.

The results are similar for both the semi-parametric and parametric analysis and they have at least four important implications for effective risk management. We found that VPE is significantly sensitive to mis-estimating correlation; as the correlation estimation error increases, the VaR mis-calculation increases. This holds for both linear and option portfolios where VaR is calculated by either variance-covariance or MC simulation methods. Therefore, the choice of a model that estimates correlation with the lowest possible error is important for risk managers.

Furthermore, we found that the sensitivity of VPE to correlation errors depends among others on the level of the 'true' correlation. As the level of the 'true' correlation decreases, this sensitivity increases. This implies that the calculated from linear portfolios VaR will be more sensitive to correlation errors for well diversified linear portfolios. In the case of option portfolios, this implies that the correlation bias effect increases as the correlation between the underlying assets decreases.

Moreover, the correlation bias effect to VaR depends also on the options moneyness level, and the time-to-maturity. The results show that the option portfolio VaR is more sensitive to correlation errors in the case that the portfolio contains short-maturity ITM options. This result is not welcomed by the risk manager, since most of the options trading activity is concentrated on short-expiry, near-the-money options.

Finally, our results indicate that the VaR sensitivity to correlation mis-estimation depends on the method that is employed to calculate VaR. Using MC simulation rather than variance-covariance methods increases this sensitivity. This reveals an extra limitation of the technique in addition to its time-consuming nature. Therefore, the accuracy of VaR results obtained from MC simulation should be verified via various backtesting methods. Furthermore, it is well known that for linear portfolios the risk manager is in principle indifferent between using variance-covariance and MC simulation methods; the calculated via MC simulation VaR converges to the calculated via variance-covariance VaR. However, since the correlation bias effect on VaR depends on the employed method, the use of MC simulation should be avoided even for small linear portfolios, if possible.

The proposed methodology can be applied to more complex portfolios containing more assets with long and short positions and more complex non-linear positions, such as option spreads. The effect of correlation errors should also be studied in the context of option pricing (e.g., basket options) and asset allocation. Moreover, the semi-parametric approach may be modified by allowing for more general structures of the correlation estimation error in the spirit of Jacquier and Jarrow (2000). These issues are well beyond the scope of the current paper, but they deserve to become topics for future research.

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