# Beating a Benchmark by Actively Managing its Components: The Case of the Dow Jones

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**Abstract**

In this paper we focus on the optimal use of a variety of predictive variables to construct actively managed strategies using the components of the Dow Jones Industrial Average, and analyze whether these outperform the index itself. Our strategies are unconditionally efficient with the portfolio weights being pre-specified functions of the predictive variables, and are thus both theoretically optimal and also relatively simple to implement. Our strategies significantly outperform not only the DJIA but also the corresponding 'fixed-weight' strategies derived from classic mean-variance theory. For example, an optimal stock-picking strategy using term spread and convexity as predictive variables has an alpha of 11.8%, while the corresponding fixed-weight strategy barely matches the index. Similarly, an optimal market-timing strategy again using the same predictor variables achieves an alpha of 10.9%, whereas the corresponding fixed-weight strategy under-performs the index by 3.4%.



JEL Classification: C31, C32, G11

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In recent years there has been a renewed focus on active portfolio management and in particular tactical asset allocation, with the search for "alpha" having taken center stage as discussed in Grinold and Kahn (1998), and Litterman (2004). This paper sets out to apply state-of-the art tools, developed in the recent academic literature, to the problem of active portfolio management. An important ingredient in this exercise is the use of predictive variables to determine portfolio allocation. In this paper we focus on the optimal use of a variety of predictive variables to construct strategies that actively manage the constituents of the Dow Jones Industrial Average, in an attempt to consistently out-perform the index. Our perspective is thus that of an active index fund manager, who is constrained to invest only in the 30 largest stocks, and does not have access to other assets (for example small firm, growth or 'glamour' stocks). Surprisingly, while this restrictive setting would seem to make it very hard to beat the index, we find that our strategies in most cases show significantly superior performance, as measured by a variety of industry-standard performance measures.

We consider eight different types of strategies which we broadly classify into optimal *stock-picking* strategies, and optimal *market-timing* strategies. While the former are constrained to invest only in the DJIA constituent stocks, strategies in the latter group are allowed to also allocate a fraction of funds to a (conditionally) risk-free Treasury bill. Within each group, we compare the performance of our actively managed strategies with that of corresponding static strategies derived from traditional portfolio theory. While all strategies are constructed to be ex-post mean-variance efficient within their class, for the actively managed ones the portfolio weights are optimal functions of the lagged predictive variables. Our strategies are both theoretically optimal and also relatively simple to implement. The strategies' performance is evaluated strictly out-of-sample, in the sense that we estimate the parameters required for their construction using only data prior to the evaluation period. Our strategies could, thus, have been implemented by a real-world portfolio manager over the time period under consideration.

To achieve optimal active management, we use a variety of predictive instruments which we broadly classify into variables reflecting market activity and liquidity, and variables capturing changes in the macro-economic environment. We examine the performance of our strategies using a variety of industry-standard performance measures, including information and Sharpe ratios, tracking errors, and Jensen's alpha relative to the index as a benchmark. To better understand the characteristics of the strategies, we also analyze their conditional performance in different regimes, for example during recessions versus periods of expansion, or periods of high versus low volatility. Finally, we also examine the cumulative returns of our strategies, compared with those of the index or a corresponding traditional mean-variance strategy, for a range of realistic investment horizons. Because the predictive variables that drive the asset allocation weights are aggregate variables like inflation or term spread, there is no obvious a priori reason to expect these variables to contain any valuable information as to the precise allocation among different stocks or industry sectors. Surprisingly, however, the empirical results unequivocally suggest that they do. While an in-depth analysis of the precise nature of the impact of such predictive variables on individual portfolio weights goes beyond the scope of this paper, research on this issue is underway by the authors and will be presented elsewhere.

It may seem that out-performing a simple static portfolio such as the Dow Jones average should be easy. However, we find that most static strategies based on classic portfolio theory in fact significantly under-perform the benchmark, particularly during recession periods. In contrast, our strategies not only out-perform the DJIA on average, but also do particularly well in recessions, thus providing a form of portfolio insurance without options. For example, in about 70% of the months in which the DJIA under-performed the return on a one-month T-bill, our strategies out-performed not only the index but also the return on the bill.

Of all the predictive instruments we consider, we find that those capturing changes in the shape of the Treasury yield curve have the most predictive power. For example, an optimal stock-picking strategy using term spread (measuring the slope of the yield curve) and convexity (measuring its curvature) achieves an overall alpha of 11.8%, while the corresponding fixed-weight strategy barely matches the index. Our strategies have information and Sharpe ratios in excess of 0.6 (almost three times that of the DJIA).

Similarly, the optimal market-timing strategy using the same variables achieves an alpha of 10.9%, whereas the corresponding fixed-weight strategy under-performs the index by 3.4%. While the performance of the market-timing strategy falls slightly short of the corresponding stock-picking strategy, the former provides a considerable amount of portfolio insurance, with an alpha of 23.1% during recessions. Moreover, while our strategy maintains its performance in periods of expansion (with an alpha of 9.5%), the corresponding fixed-weight strategy under-performs the index by 5.5% in such periods.

By construction, our strategies are designed for investors with medium-term investment horizon; their true potential becomes apparent in time-aggregation. At the monthly frequency, there are still some periods in which our strategies under-perform the benchmark. However, when we consider the cumulative returns over any five-year sub-period, we find only one (out of 23) instance in which our strategy significantly under-performs the index. Moreover, while the DJIA under-performs the return on risk-free T-bills in 8 of the 23 five-year periods, our optimal strategy yields a positive excess return in every one of these periods. Our results thus provide evidence that return predictability used optimally can produce considerable economic gains, even when the asset universe is severely constrained.

In order to better understand how our strategies achieve their superior performance, we analyze their investment style using the Fama-French three-factor model. While both the fixed-weight as well as the optimal strategies deviate substantially from tracking the benchmark, only the optimal active management turns this disassociation into superior performance. In fact, although the asset universe consists predominantly of large value stocks (the DJIA constituents), the active management moves the behavior of our strategies closer towards that of growth stocks, particularly for the market-timing strategies. Analyzing the portfolio weights, we find that the market-timing strategies move funds out of the stock market and into the risk-free T-bill in times when the index is expected to perform poorly and vice versa, as should be expected.

Other predictive instruments we consider include realized index volatility, aggregate turnover in the DJIA constituents, and other macro-economic indicators such as unexpected shocks to inflation, changes in unemployment and aggregate consumer spending, as well as average credit yield spreads. We find that, while each of these variables on their own improves the performance of our strategies marginally, none comes even close to matching term spread and convexity. In fact, adding any of these variables to the latter slightly reduces performance. In other words, in the context of portfolio formation most of the information conveyed by these additional instruments is subsumed by changes in the shape of the yield curve, which could be interpreted as describing changes in monetary policy. Interestingly, it is not the most recent changes that matter most; instead the maximum level of predictive power is achieved at a three-month lag. This seems to indicate that it takes on average several months for changes in monetary policy to take effect in the economy.

The remainder of this paper is organized as follows. We first briefly describe our methodology, and then summarize the results of our empirical analysis. A detailed description of the construction of our optimal strategies and the data used can be found in the appendix.

# Data and Methodology

For our study, we consider monthly returns on the DJIA and its constituent stocks, covering the sample period from August 1976 to October 2004. The start date was chosen to coincide with one of the dates at which the index composition changed. During our sample period, there were 8 structural changes to the index, with some of its components being removed and replaced by others. This leaves us with 9 sub-sample periods between the re-balancing dates. The average length of the sub-sample periods is about 40 months, with the shortest (from April to October 2004) spanning only 7 months and the longest (from May 1991 to March 1997) extending to almost 6 years. A detailed description of the index composition and the re-balancing events can be found in Appendix B.

For each of the 9 sub-sample periods, we estimate our predictive model (see Appendix A for details), using return data on the 30 stocks that made up the index during that period, and the chosen predictive variables. Based on the estimated model parameters, we then construct the portfolio weights of our optimally managed strategies as functions of the predictive instruments. To avoid any look-ahead bias, we use for the estimation only data preceding the beginning of the respective period. We then 'run' the strategies using stock return and lagged instrument values within the period, and compute their returns. This procedure guarantees that the strategies could have been implemented by a real-world portfolio manager, as the portfolio allocations are based only on information available at the time they are chosen. For each strategy, we then concatenate the return series for all sub-sample periods, to obtain a time series of returns that spans the entire sample period from 1976 to 2004. These time series are the basis for our ex-post performance analysis.

In addition to the simple in-sample/out-of-sample estimation, we also implement a rolling-window method. In this case, we begin as before by estimating the model using data that predates the first sub-period. However, we then construct the strategies only for the next month ahead. We then re-estimate the model using the additional data point, extend the strategy another month ahead, and so on.

Strategies

For each (set of) predictive instrument(s), we construct eight different strategies which we broadly classify into (a) optimal *stock-picking* strategies, and (b) optimal *market-timing* strategies. Strategies in group (a) are constrained to invest only in the DJIA constituent stocks, while those in group (b) are allowed to also allocate a fraction of funds to a (conditionally) risk-free Treasury bill. Within each group, we consider two types of efficient strategies, (a) those that are designed to minimize volatility for a given target average return, and (b) those that maximize return for a given target volatility. For comparison, the target return and volatility are chosen to match those of the DJIA over the sample period under consideration. For each type, we compare the performance of (i) the static strategy derived from traditional mean-variance theory, and (ii) the corresponding actively managed strategy that exploits return predictability optimally. While all strategies are constructed to be ex-post mean-variance efficient within their class, for those in group (ii) the portfolio weights are functions of the lagged predictive variables. A detailed description of their construction can be found in Appendix A.

### Instruments

We group the predictive instruments used into two broad categories, (a) variables that capture market activity and liquidity, and (b) those that describe the state of the overall economy. Our choice of instruments is motivated by recent empirical evidence which documents the significant ability of certain variables to predict the distribution of asset returns. More specifically, in group (a) we include (de-trended) turnover ratio[1] as a measure of market liquidity, and the monthly realized variance of the index, computed as the sum of squared daily returns throughout the month. In group (b) we include term spread (measuring the slope of the Treasury yield curve), convexity (measuring its curvature), average credit yield spread, as well as macro-economic indicators such as unexpected shocks to inflation, changes in unemployment and aggregate consumer spending[2]. A detailed description of the variables used and the way in which they are constructed is given in Appendix B.

### Performance

To assess the performance of the strategies, we compute several ex-post performance statistics, including Sharpe and information ratios. As the main focus of this study is the outperformance of the market benchmark, we place particular emphasis on Jensen's alpha, both relative to the index itself, as well as relative to the Fama-French three-factor model. In order to better understand how these strategies perform in different regimes, we also construct regime-dependent alphas. Specifically, we consider recession versus expansion, high versus low volatility, as well as high and low trading activity (see Appendix B for details).

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[1]Measures of liquidity have been the focus of recent research, and this study is the first to use such a measure as a predictive instrument.

[2]In recent work, Abhyankar, Basu, and Stremme (2005a) find that some of these variables have considerable predictive power for asset returns.

# Results

We first report the results for the stock-picking strategies (in the absence of a risk-free asset), and then move on to the market-timing strategies.

## Stock-Picking Strategies

We first focus on strategies that are constrained to invest only in the 30 component stocks of the index. The results for this case, for two different sets of instruments, are reported in Panel A of Tables 2, 3 and 4, respectively.

### Volatility and Turnover

Table 2 reports the results in the case where the predictive instruments are realized index variance (RV), and the (de-trended) turnover ratio (TO). Our actively managed maximum-return strategy not only out-performs the DJIA (with an overall alpha of 4.98%), but also beats the corresponding fixed-weight strategy which just barely matches the index performance (with an alpha of less than 1%). In fact, the optimal strategy has slightly lower risk (21.8%) than the fixed-weight portfolio (22.0%), but a considerably higher average return (13.7% versus 9.0%). Conversely, while the average return of the fixed-weight portfolio is almost identical to that of the DJIA, its variance is about 50% higher. Our optimally managed strategy has a Sharpe ratio of 0.31, compared to only 0.22 for the Dow Jones.

Breaking down the performance according to the business cycle, we find that the active strategy shows consistent out-performance in both recession and expansion (with an alpha in recession periods of 7.5%, and 4.7% in expansion periods), while the fixed-weight strategy in fact slightly under-performs the index in expansion periods. Also interesting is the 'style' analysis: while both fixed-weight and active strategy are clearly (and unsurprisingly) biased towards large stock (with similar coefficients of about −0.39 and −0.37 for the SMB factor), the active portfolio behaves much more like a 'value' strategy (with an HML coefficient of

0.23, compared to 0.16 for the DJIA itself) than the fixed-weight strategy, which does not have a clear bias towards either growth or value stocks. Finally, our active strategy outperforms even when size and book-to-market effects have been corrected for (with an alpha of 2.38% in the Fama-French model), while the fixed-weight strategy in fact under-performs slightly.

The overall alpha of the minimum-variance strategy (results not reported in the table) is 4.9%, almost identical to the maximum return strategy. However it under-performs during recessions with a recession alpha of $-4.8\%$ and outperforms during expansion periods with an alpha of 6.9%.

Term Spread

Tables 3 and 4 report the results in the case where the predictive instruments are the term spread (TSPR), and term spread plus convexity (CONV), respectively. We first focus on the performance of term spread as a predictive variable (Table 3). The actively managed maximum-return strategy outperforms both the DJIA as well as the corresponding fixed-weight strategy along all the dimensions considered. The overall alpha is 9.97% with virtually the same level of risk as the fixed weight strategy (21.9% compared to 22.0%) but more than double the return (19.2% compared to 9.0%). Our actively managed strategy has a Sharpe ratio of 0.53, more than twice that of the Dow (0.22), and its information ratio is 0.51.

This strategy seems to perform better in recession periods with a recession alpha of 13.0% but, unlike the corresponding fixed-weight strategy, also outperforms the index in expansions with an expansion alpha of 9.6%. Another interesting feature of this strategy is that it performs better in high volatility and high activity periods (with alphas of 10.5 and 13.8% respectively) than low volatility and low activity periods (alphas of 8.8 and 6.1% respectively), which is an indication that our strategy responds actively to information arrival.

The minimum-variance strategy (results not reported in the table) also beats the DJIA with an overall alpha of 4.1% but its performance over the business cycle is very different to the maximum-return strategy, as it under-performs during recession (with a recession alpha of

−5.4%). It also performs better during low volatility periods (alpha of 9.7%) than high volatility periods (alpha of 1.8%), in contrast to the maximum-return strategy that does the opposite. This strategy achieves Sharpe and information ratios of 0.37 and 0.31, which are considerably lower than the maximum return strategy.

### Convexity

Next, we investigate the predictive ability of convexity (CONV), which measures the curvature of the Treasury yield curve (see Appendix B for details). While this variable on its own does not achieve the performance of the term spread, using both variables together considerably improves the performance of our strategies (Table 4).

Again, the actively managed maximum-return strategy outperforms both the DJIA as well as the corresponding fixed-weight strategy. The overall alpha is 11.8% with a slightly lower level of risk as the fixed weight strategy (21.6% compared to 22.0%), but more than double the return (21.4% compared to 9.0%). Our actively managed strategy has a Sharpe ratio of 0.63, almost triple that of the Dow (0.22), and its information ratio is 0.63.

All other aspects are qualitatively similar to the case with only term spread as predictive instrument, with a slight improvement of performance across all measures considered. The only difference is that the optimal strategy now shows a slight bias towards value stocks (with an HML coefficient of 0.13 in the Fama-French model). The results for the corresponding minimum-variance strategy are also very similar.

### Other Instruments

We also examined the performance of our strategies using average credit yield spread (CSPR), unexpected shocks to inflation (INF), and changes in unemployment (UEGR) and aggregate consumer spending (CGR). We find that, while each of these variables on their own improves the performance of our strategies marginally, none comes even close to matching term spread and convexity. In fact, adding any of these variables to the latter slightly reduces performance. In other words, most of the information conveyed by these additional instruments is subsumed by changes in the shape of the yield curve, which could be interpreted as describing changes in monetary policy. Interestingly, it is not the most recent changes that matter most; instead the maximum level of predictive power is achieved at a three-month lag. This seems to indicate that it takes on average several months for changes in monetary policy to take effect in the economy.

Thus, our optimal stock-picking strategies significantly outperform not only the DJIA, but also the corresponding fixed-weight strategies. The term structure variables (TSPR and CONV) perform best as predictors, and strategies using these variables perform particularly well during periods of recession. The active management manifests itself in the fact that our strategies show better performance in periods of high volatility and high trading activity.

## Market-Timing Strategies

We now consider the case in which the strategies are allowed to allocate funds between the 30 Dow Jones component stocks and a 1-month Treasury bill. Note that, while the returns on T-bills of course vary over time and hence are risky ex-post, the 1-month-return at the time when portfolio decisions are made is known and thus (conditionally) risk-free. We use this fact in the construction of our optimal strategies (see Appendix A).

### Volatility and Turnover

Panel B of Table 2 reports the results for the case where the predictive instruments are realized variance (RV), and the (de-trended) turnover ratio (TO). In the presence of a (conditionally) risk-free asset, our actively managed maximum-return strategy marginally out-performs the DJIA (with an overall alpha of 1.46%), while the fixed-weight strategy under-performs by 3.39%. This is largely driven by the fact that, while both strategies show good performance in recessions (with recession alphas of 12.12% and 14.52%, respectively), our strategy tracks the index almost perfectly in boom periods (with an alpha of 0.17%), whereas the fixed-weight strategy under-performs considerably (with an alpha of −5.47%) in these periods. In other words, our optimally managed market-timing strategy provides defacto portfolio insurance in the sense that it matches market performance when the economy is growing, while significantly out-performing the market in recessions. In fact, while the DJIA earned on average 2% less than T-bills in recessions, our strategy achieves an average excess return of 11% over T-bills.

### Term Spread and Convexity

The results for the case of term spread (TSPR) and convexity (CONV) as predictive instrument are reported in Panel B of Tables 3 and 4.

Using term spread alone, the maximum-return strategy has an overall alpha of 8.6%, achieving Sharpe and information ratios of 0.43 and 0.41, respectively. The performance over the business cycle, while qualitatively similar to the corresponding stock-picking strategy, seems to provide greater portfolio insurance (with a recession alpha of 18.1%). The difference in performance between high and low volatility and activity regimes is not as pronounced as that for the respective stock-picking strategy.

Note that, in the presence of a risk-free asset, this optimal strategy behaves more like a 'growth' than a value strategy (with an HML coefficient of $-0.24$). The minimum-variance strategy behaves much more like an index-tracking strategy, with the overall as well as the regime-dependent alphas all virtually zero, and a negligible tracking error.

The effect of including convexity in the set of predictive instruments is very similar to the stock-picking strategies, with the overall alpha increasing to 10.9%, and Sharpe and information ratios of 0.56 and 0.54, respectively. However, the difference in performance between the recession and boom, and high and low volatility or activity regimes is much more pronounced.

Figure 1 shows the cumulative returns over the entire sample period from 1976 to 2004. While the Dow Jones itself under-performs the performance of Treasury bills (depicted as the smooth solid line) for the first 20 years, our optimally managed strategies shows superior performance from the start. While an investment in the DJIA would have earned a total of only $10 for each $1 invested in 1976, the same amount invested in the optimal market-timing strategy would have grown to almost $100.

Figure 2 shows the (annualized) cumulative returns of our strategy (vertical axis), graphed against those of the DJIA (horizontal axis), over any of the 23 five-year sub-periods in the sample period. While there is only one instance in which our strategy significantly under-performs the index, there are 8 periods in which the Dow not even matches the performance of an investment in Treasury bills. On the other hand, there is not a single period in which our strategy under-performs the T-bill, but there are 10 periods in which our strategy out-performs the index by 10% or more.

Our findings are similar in spirit to those in Cochrane (1999), where it is shown that an optimal market-timing strategy that utilizes return predictability can achieve Sharpe ratios that are almost double that of a buy-and-hold investor, and also confirms the findings in Wagner (1997). Our results are also consistent with Ferson and Schadt (1996), who find that mutual funds, while unable to time the market in a traditional fixed-weight setting, are able to do so when the parameters of the model are allowed to be time-varying.

## Summary

This paper sets out to illustrate in a realistic setting the value optimally using return predictability in active portfolio management. To this end, we construct a variety of ex-post efficient dynamic trading strategies, with portfolio weights that are optimally chosen functions of one or more (lagged) predictor variables. We assess the out-of-sample performance of these strategies by computing a range of commonly used performance measures, both in absolute terms as well as relative to the benchmark itself.

Overall, both the optimal stock-picking as well as market-timing strategies significantly out-perform not only the benchmark, but also the corresponding fixed-weight strategies. While stock-picking strategies show slightly higher overall performance, market-timing strategies provide better portfolio insurance in recessions. Of all the predictive instruments we tested, term spread and convexity perform best by a wide margin. Other variables, while achieving marginal performance gains on their own, in fact reduce performance slightly when used in connection with the former. This effect may be due to noisy data, with our estimation procedure picking up spurious predictive relationships to which our strategies respond, thus generating additional volatility that is not offset by corresponding gains in return.

Interestingly, the term structure variables achieve their maximal predictive potential at the 3-month lag, indicating that changes in monetary policy take a few months to take effect in the economy. Both the fixed-weight as well as optimal strategies deviate substantially from tracking the benchmark, unsurprisingly more so for market-timing strategies. However, only the optimal active management turns this disassociation into superior performance. In fact, although the asset universe consists predominantly of large value stocks (the DJIA constituents), the active management moves the behavior of the market-timing strategy closer towards that of growth stocks.

# Appendix A: Optimally Managed Strategies

The optimal use of conditioning information in the formation of portfolios was first studied, in a very theoretical framework, by Hansen and Richard (1987). More recently, their results were operationalized by Ferson and Siegel (2001), and Abhyankar, Basu, and Stremme (2005b). The objective is to characterize the weights, as functions of the conditioning instruments, of managed portfolio strategies that are unconditionally mean-variance efficient.

We consider any one given investment period (i.e. month). Denote by time $t-1$ the beginning of the period, and the end by $t$. At time $t-1$, the investor allocates funds across the 30 component stocks of the DJIA. We denote the gross return (per dollar invested) of the $k$-th stock by $r_t^k$, and by $\tilde{R}_t := (\, r_t^1 \, \ldots \, r_t^{30} \,)'$ the vector of stock returns. For the moment, we assume that no risk-free asset is traded.

While returns are realized at time $t$, the portfolio allocation will depend on the information available at time $t-1$. We assume that this information is summarized entirely by a vector $y_{t-1} = \left( y_{t-1}^1 \, \ldots \, y_{t-1}^m \right)'$ of lagged *conditioning instruments*, variables observable at time $t-1$ that contain information about the distribution of stock returns over the next period[3]. Finally, we denote by $E_{t-1}(\,\cdot\,)$ the conditional expectation with respect to $y_{t-1}$.

An *actively managed portfolio* can then be described by a vector $\theta_{t-1} = (\, \theta_{t-1}^1 \, \ldots \, \theta_{t-1}^{30} \,)'$ of *weights*, where the $\theta_{t-1}^k$ are functions of the conditioning information, $y_{t-1}$. Because the weights represent the fraction of the investor's wealth invested in each of the stocks, we require that the weights add up to 100%, i.e. $e' \, \theta_{t-1} \equiv 1$, where $e$ is a vector of 'ones'. The *return* on such a portfolio is then given by $\tilde{R}_t' \, \theta_{t-1}$. We denote by $R_t$ the set of all returns that can are attainable by such managed strategies[4].

[3]These variables include indicators of market activity such as realized variance (RV) and turnover ratio (TO), and economic indicators such as term-spreads (TSPR) or inflation shocks (INF). See [REF] for a detailed description of the instruments used in this study.

[4]Note that, in contrast to the fixed-weight case without conditioning information, the space of *managed*

Efficient Portfolio Weights

For given unconditional expected return $m$, the objective is to construct the dynamically managed portfolio $r_t^*(m)$ that has minimal variance among all portfolios with mean $m$. To begin with, we define the conditional moments of the base asset returns as,

$$\mu_{t-1} = E_{t-1}\big(\tilde{R}_t\big), \qquad \text{and} \qquad \Lambda_{t-1} = E_{t-1}\big(\tilde{R}_t \cdot \tilde{R}_t'\big). \tag{1}$$

In other words, returns can be written as $\tilde{R}_t = \mu_{t-1} + \varepsilon_t$, where $\mu_{t-1}$ is the conditional expectation of returns given conditioning information, and $\varepsilon_t$ is the residual disturbance with variance-covariance matrix $\Sigma_{t-1} = \Lambda_{t-1} - \mu_{t-1}\mu_{t-1}'$. This is the formulation of the model with conditioning information used in Ferson and Siegel (2001)[5]. Finally, we set

$$A_{t-1} = e'\Lambda_{t-1}^{-1}e, \qquad B_{t-1} = \mu_{t-1}'\Lambda_{t-1}^{-1}e, \qquad D_{t-1} = \mu_{t-1}'\Lambda_{t-1}^{-1}\mu_{t-1} \tag{2}$$

These are the conditional versions of the 'efficient set' constants from classic mean-variance theory. We choose this notation in order to highlight the structural similarities between the UE and GHT bounds, and to facilitate a direct comparison.

Abhyankar, Basu, and Stremme (2005b) show that the unconditionally efficient portfolio $r_t^*(m) \in R_t$ for given unconditional mean $m$ can be written as $r_t^*(m) = \tilde{R}_t'\theta_{t-1}$, where

$$\theta_{t-1} = \Lambda_{t-1}^{-1}\Big( \frac{1 - wB_{t-1}}{A_{t-1}}\, e + w\, \mu_{t-1} \Big), \tag{3}$$

where $w$ is a constant, given by $w = (\, m - E(\, B_{t-1}/A_{t-1}\,)\,)/E(\, D_{t-1} - B_{t-1}^2/A_{t-1}\,)$. It is now easy to show that the variance of the efficient return is given by,

$$\sigma^2(\, r_t^*(m)\,) = E(\, 1/A_{t-1}\,) + \frac{(\, m - E(\, B_{t-1}/A_{t-1}\,)\,)^2}{E(\, D_{t-1} - B_{t-1}^2/A_{t-1}\,)} - m^2. \tag{4}$$

Hence, to find the portfolio with maximal return for given target variance $\bar{\sigma}^2$, one simply needs to find the larger root $m$ of the quadratic equation $\sigma^2(\, r_t^*(m)\,) = \bar{\sigma}^2$.

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pay-offs is infinite-dimensional even when there is only a finite number of base assets.

[5]Note however that our notation differs slightly from that used in Ferson and Siegel (2001), who define $\Lambda_{t-1}$ to be the *inverse* of the conditional second-moment matrix.

### Modeling Return Predictability

To construct the optimal portfolio strategies derived in the preceding section, we need to estimate the conditional moments $\mu_{t-1}$ and $\Lambda_{t-1}$ as functions of the conditioning information. Although the results in Abhyankar, Basu, and Stremme (2005b) are not restricted to a linear specification, for the purpose of this paper we will assume that the relation between returns and conditioning instruments is given by a linear predictive model of the form,

$$\tilde{R}_t = \mu_0 + B \cdot y_{t-1} + \varepsilon_t, \tag{5}$$

where $y_{t-1}$ is the vector of (lagged) conditioning variables. To estimate this model by OLS, we assume furthermore that the residuals $\varepsilon_t$ are independent of $y_{t-1}$, serially independently and identically distributed with $E_{t-1}(\varepsilon_t) = 0$. The vector of conditional expected returns in this case becomes, $\mu_{t-1} = \mu_0 + B \cdot y_{t-1}$. The independence assumption implies that the conditional variance-covariance matrix does not depend on $y_{t-1}$, and we write $\Sigma$ instead of $\Sigma_{t-1}$. Finally, the matrix of second moments can then be written as $\Lambda_{t-1} = \Sigma + \mu_{t-1}\mu'_{t-1}$.

### Incorporating a Risk-Free Asset

Incorporating a risk-free asset is not as straight-forward as it may seem. Traditional mean-variance theory assumes that the risk-free rate is constant and known in advance for all periods. In reality however, this is not the case. Instead, we consider here a *conditionally* risk-free rate.

More specifically, denote by $r_t^F$ the return on a T-bill over the period from time $t-1$ to time $t$. While $r_t^F$ may change over time, its value is known with certainty at time $t-1$. To incorporate this information, we first estimate the predictive model as in (5). Then, we augment the vector of asset returns and instruments to include the risk-free asset, $\tilde{R}_t^+ = (r_t^F, \tilde{R}_t)'$, and $y_{t-1}^+ = (r_t^F, y_{t-1})'$. Note that we did not lag the risk-free rate, as it is known at time $t-1$. The predictive equation, taking into account the fact that the risk-free rate is known at the beginning of the month, can now be written as

$$\tilde{R}_t^+ = \mu_0^+ + B^+ \cdot y_{t-1}^+ + \varepsilon_t^+, \tag{6}$$

where $\mu_0^+ = (0, \mu_0)'$, $\varepsilon_t^+ = (0, \varepsilon_t)'$, and the coefficient matrix becomes

$$B^+ = \begin{pmatrix} 1 & 0 \\ 0 & B \end{pmatrix} \tag{7}$$

This way, the model incorporates the information about the one-period ahead risk-free return, without assuming an unconditionally constant risk-free rate. The efficient portfolio weights can then be computed as in (3), replacing all variables by their respective augmented versions. Note that, as a consequence of the above construction, the variance-covariance matrix $\Sigma^+$ of the augmented residuals $\varepsilon_t^+$ is now singular, as the first row and column are zero. However, this does not pose a problem as the efficient weights are formulated in terms of the matrix $\Lambda_{t-1}^+$ of second moments which is, in contrast to $\Sigma^+$, invertible.

# Data Description

Daily data for all the DJIA constituents over the period July 2, 1962 through December 31, 2004 are taken from the CRSP dataset. The following variables are selected: (a) holding period return, i.e. return adjusted for dividend payments and stock splits, (b) volume, i.e. the total number of shares of a stock sold on a given day, and (c) shares outstanding, i.e. the total number of publicly held shares. Table 1 shows the re-balancing events for the index.

### Instruments

We construct two groups of predictive instruments, one containing market-based variables derived from the DJIA data, and a second containing economic indicators. The market-based instruments are (a) monthly realized variance (RV), the sum of squared daily holding period returns on the index, and (b) turnover ratio (TO), defined as the daily volume divided by number of shares outstanding, averaged across stocks and over the month. We found a clear linear trend in log turnover, with a regime break around October 1987. We thus estimated the linear trend in the log instrument for the two periods before and after the break point, and used the residual deviation from the trend as our predictive instrument.

For the second group of variables we choose (a) term spread (TSPR), defined as the difference between the yield of 10-year Treasury bonds and the 1-year T-bill rate, (b) convexity (CONV), defined as twice the 5-year yield minus the 10-year and 1-year yields. Both variables were constructed from data published by the Federal Reserve Bank of St. Louis[6]. We also consider (c) credit yield spread, defined as the difference in 10-year yield between AAA-rates corporate and government bonds, which we obtained from Datastream. Finally, we include macro-economic indicators measuring (d) unexpected shocks to inflation, and changes in (e) unemployment and (f) aggregate consumer spending, all constructed from data published by the Federal Reserve Bank of St. Louis.

### Performance Measures

We compute a variety of ex-post performance measures from the time series of returns on the strategies and the benchmark. In particular, (a) Jensen's alpha is obtained as the intercept of the regression of excess (over T-bill) returns on the strategy on the excess returns on the DJIA benchmark, where the (b) tracking error is defined as the residual volatility in this regression. The (c) information ratio is calculated as the strategy's alpha (a), divided by the tracking error (b). Finally, the style indicators are computed by regressing the strategy's excess returns on the excess return on a market index, and the two Fama-French factors SMB ('small minus big') and HML ('high minus low'). The coefficients on the latter two factors capture the ex-post investment style of the strategy: a positive coefficient on SMB indicates that the strategy's behavior is biased towards small stocks, while a positive coefficient on HML indicates a bias towards value stocks (those with high book-to-market ratios).

### Regimes

To further analyze the performance of our strategies in different regimes, we construct indicator variables separating (a) periods of recession and expansion, (b) periods of low and high volatility, and (c) periods of low and high market turnover, respectively. For (a), we

[6] `http://research.stlouisfed.org/fred2/`.

use the recession index constructed ex-post from the macro-economic indicators published by the the National Bureau of Economic Research (NBER)[7]. The market activity variables (b) and (c) were constructed from the respective instruments (RV and TO), using a high-low switching band around their respective mean. In other words, the indicator is set to 'high' when the respective instrument value passes through the upper limit, and set to 'low' when it falls below the lower limit. This procedure was chosen to avoid the indicator switching between regimes even for very small changes in the instrument. All regime indicators take on the values 1 and 0 to indicate recession and boom, or high versus low volatility and turnover. To obtain conditional performance alphas, the constant in the regression of portfolio returns on the benchmark is replaced by the regime indicator and a second dummy variable which takes on the value 1 when the indicator is zero and vice versa.

[7] http://www.nber.org.

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| | Date | Added to DJIA | Removed from DJIA |
|---|---|---|---|
| 1. | < 1 Jun 1959 | Alcoa Inc. | |
| 2. | | Du Pont | |
| 3. | | Exxon Mobil | |
| 4. | | General Electric Company | |
| 5. | | General Motors Corporation | |
| 6. | | Honeywell International | |
| 7. | | Procter & Gamble Company | |
| 8. | | United Technologies Corp | |
| 9. | 9 Aug 1976 | 3M Company | Anaconda Copper |
| 10. | 29 Jun 1979 | International Business Machines | Chrysler |
| 11. | | Merck | Esmark |
| 12. | 30 Aug 1982 | American Express Company | Johns-Manville |
| 13. | 30 Oct 1985 | Altria Group | American Tobacco B |
| 14. | | McDonalds Corporation | General Foods |
| 15. | 12 Mar 1987 | Coca-Cola | Owens-Illinois Glass |
| 16. | | Boeing Company | Inco |
| 17. | 6 May 1991 | Caterpillar Incorporated | Navistar International Corp. |
| 18. | | Walt Disney Company | USX Corporation |
| 19. | | J.P. Morgan Chase | Primerica Corporation |
| 20. | 17 Mar 1997 | Citigroup Incorporated | Westinghouse Electric |
| 21. | | Hewlett-Packard Company | Texaco Incorporated |
| 22. | | Johnson & Johnson | Bethlehem Steel |
| 23. | | Wal-Mart Stores Incorporated | Woolworth |
| 24. | 1 Nov 1999 | Microsoft Corporation | Chevron |
| 25. | | Intel Corporation | Goodyear |
| 26. | | SBC Communications | Union Carbide |
| 27. | | Home Depot Incorporated | Sears Roebuck & Company |
| 28. | 8 Apr 2004 | American Intl. Group Inc | AT&T Corporation |
| 29. | | Pfizer Incorporated | Eastman Kodak Company |
| 30. | | Verizon Communications Inc | International Paper Company |

**Table 1: DJIA Constituents 1959–2005**

This table lists the changes in composition of the Dow Jones Industrial Average (DJIA) throughout the sample period under consideration. Note that there were also several name changes and mergers in this period, which we do not list here.

| | Panel A: Stock Picking Strategies (no risk-free asset) | | Panel B: Market Timing Strategies (with risk-free asset) | |
|---|---|---|---|---|
| | fixed-weight | optimal | fixed-weight | optimal |
| Expected Return | 9.00% | 13.68% | 3.78% | 8.43% |
| Return Volatility | 22.04% | 21.87% | 22.70% | 20.01% |
| Sharpe Ratio | 0.1174 | 0.3141 | −0.1029 | 0.1038 |
| | Performance Relative to Benchmark | | | |
| Benchmark Beta | 0.5886 | 0.5862 | 0.3282 | 0.1864 |
| Jensen's Alpha | 0.60% | 4.98% | −3.39% | 1.46% |
| Tracking Error | 20.11% | 19.87% | 22.13% | 19.79% |
| Information Ratio | 0.0299 | 0.2507 | −0.1531 | 0.0735 |
| | Regime-Dependent Performance (Conditional Alpha) | | | |
| Recession | 13.08% | 7.54% | 14.52% | 12.12% |
| Expansion | −0.89% | 4.66% | −5.47% | 0.17% |
| High Volatility | 0.53% | 3.07% | −2.57% | −1.09% |
| Low Volatility | 0.77% | 9.68% | −5.32% | 7.78% |
| High Activity | 4.22% | 6.68% | −0.98% | 1.79% |
| Low Activity | −3.08% | 3.23% | −5.86% | 1.10% |
| | Performance Relative to Fama-French 3-Factor Model | | | |
| Beta (Market Return) | 0.5759 | 0.6558 | 0.2522 | 0.1695 |
| Beta (SMB) | −0.3902 | −0.3737 | −0.2774 | −0.2103 |
| Beta (HML) | −0.0315 | 0.2250 | −0.2765 | −0.0716 |
| Alpha | −0.08% | 2.38% | −1.91% | 1.90% |

**Table 2: Maximum-Return Strategies (Market Indicators)**
This table reports various ex-post performance measures for the optimal and fixed-weight maximum return strategies. The predictive variables are realized variance (RV) and de-trended turnover ratio (TO), both of which are described in Appendix B. The maximum return strategy has target volatility equal to that of the DJIA over the 1976-2004 period. The weights of the optimal strategy are given in Appendix A, while the fixed-weight strategy is constructed using standard mean-variance theory. The regime-indicators used to compute conditional alphas are constructed as described in Appendix B.

| | Panel A: Stock Picking Strategies (no risk-free asset) | | Panel B: Market Timing Strategies (with risk-free asset) | |
|---|---|---|---|---|
| | fixed-weight | optimal | fixed-weight | optimal |
| Expected Return | 9.00% | 19.22% | 3.78% | 16.54% |
| Return Volatility | 22.04% | 21.93% | 22.70% | 21.68% |
| Sharpe Ratio | 0.1174 | 0.5330 | −0.1029 | 0.4329 |
| | Performance Relative to Benchmark | | | |
| Benchmark Beta | 0.5886 | 0.6281 | 0.3282 | 0.3151 |
| Jensen's Alpha | 0.60% | 9.97% | −3.39% | 8.62% |
| Tracking Error | 20.11% | 21.88% | 22.13% | 21.08% |
| Information Ratio | 0.0299 | 0.5074 | −0.1531 | 0.4090 |
| | Regime-Dependent Performance (Conditional Alpha) | | | |
| Recession | 13.08% | 13.04% | 14.52% | 18.14% |
| Expansion | −0.89% | 9.59% | −5.47% | 7.46% |
| High Volatility | 0.53% | 10.46% | −2.57% | 8.88% |
| Low Volatility | 0.77% | 8.82% | −5.32% | 8.01% |
| High Activity | 4.22% | 13.76% | −0.98% | 10.58% |
| Low Activity | −3.08% | 6.11% | −5.86% | 6.60% |
| | Performance Relative to Fama-French 3-Factor Model | | | |
| Beta (Market Return) | 0.5759 | 0.6421 | 0.2522 | 0.2485 |
| Beta (SMB) | −0.3902 | −0.2400 | −0.2774 | −0.1370 |
| Beta (HML) | −0.0315 | 0.0611 | −0.2765 | −0.2393 |
| Alpha | −0.08% | 7.91% | −1.91% | 9.59% |

**Table 3: Maximum-Return Strategies (Term Spread)**

This table show the ex-post performance of the optimal and fixed-weight maximum return strategies in the case where the predictive instrument is the term spread (TSPR). The maximum-return strategy has target volatility equal to that of the DJIA over the 1976-2004 period. The weights of the optimal strategy are given in Appendix A, while the fixed-weight strategy is constructed using standard mean-variance theory. The regime-indicators are constructed as described in Appendix B.

| | Panel A: Stock Picking Strategies (no risk-free asset) | | Panel B: Market Timing Strategies (with risk-free asset) | |
|---|---|---|---|---|
| | fixed-weight | optimal | fixed-weight | optimal |
| Expected Return | 9.00% | 21.35% | 3.78% | 19.04% |
| Return Volatility | 22.04% | 21.55% | 22.70% | 20.82% |
| Sharpe Ratio | 0.1174 | 0.6262 | −0.1029 | 0.5546 |
| | Performance Relative to Benchmark | | | |
| Benchmark Beta | 0.5886 | 0.6804 | 0.3282 | 0.3225 |
| Jensen's Alpha | 0.60% | 11.76% | −3.39% | 10.94% |
| Tracking Error | 20.11% | 18.81% | 22.13% | 20.17% |
| Information Ratio | 0.0299 | 0.6251 | −0.1531 | 0.5424 |
| | Regime-Dependent Performance (Conditional Alpha) | | | |
| Recession | 13.08% | 16.37% | 14.52% | 23.11% |
| Expansion | −0.89% | 11.19% | −5.47% | 9.48% |
| High Volatility | 0.53% | 12.78% | −2.57% | 12.14% |
| Low Volatility | 0.77% | 9.35% | −5.32% | 8.12% |
| High Activity | 4.22% | 16.52% | −0.98% | 13.87% |
| Low Activity | −3.08% | 6.95% | −5.86% | 7.95% |
| | Performance Relative to Fama-French 3-Factor Model | | | |
| Beta (Market Return) | 0.5759 | 0.6836 | 0.2522 | 0.2695 |
| Beta (SMB) | −0.3902 | −0.1997 | −0.2774 | −0.0463 |
| Beta (HML) | −0.0315 | 0.1305 | −0.2765 | −0.0979 |
| Alpha | −0.08% | 9.05% | −1.91% | 10.74% |

**Table 4: Maximum-Return Strategies (Term Spread and Convexity)**
This table show the ex-post performance of the optimal and fixed-weight maximum return strategies in the case where the predictive instruments are the term spread (TSPR) and convexity (CONV). The maximum-return strategy has target volatility equal to that of the Dow Jones Industrial Average (DJIA) over the 1976-2004 period. The weights of the optimal strategy are given in Appendix A, while the fixed-weight strategy is constructed using standard mean-variance theory. The regime-indicators are constructed as described in Appendix B.

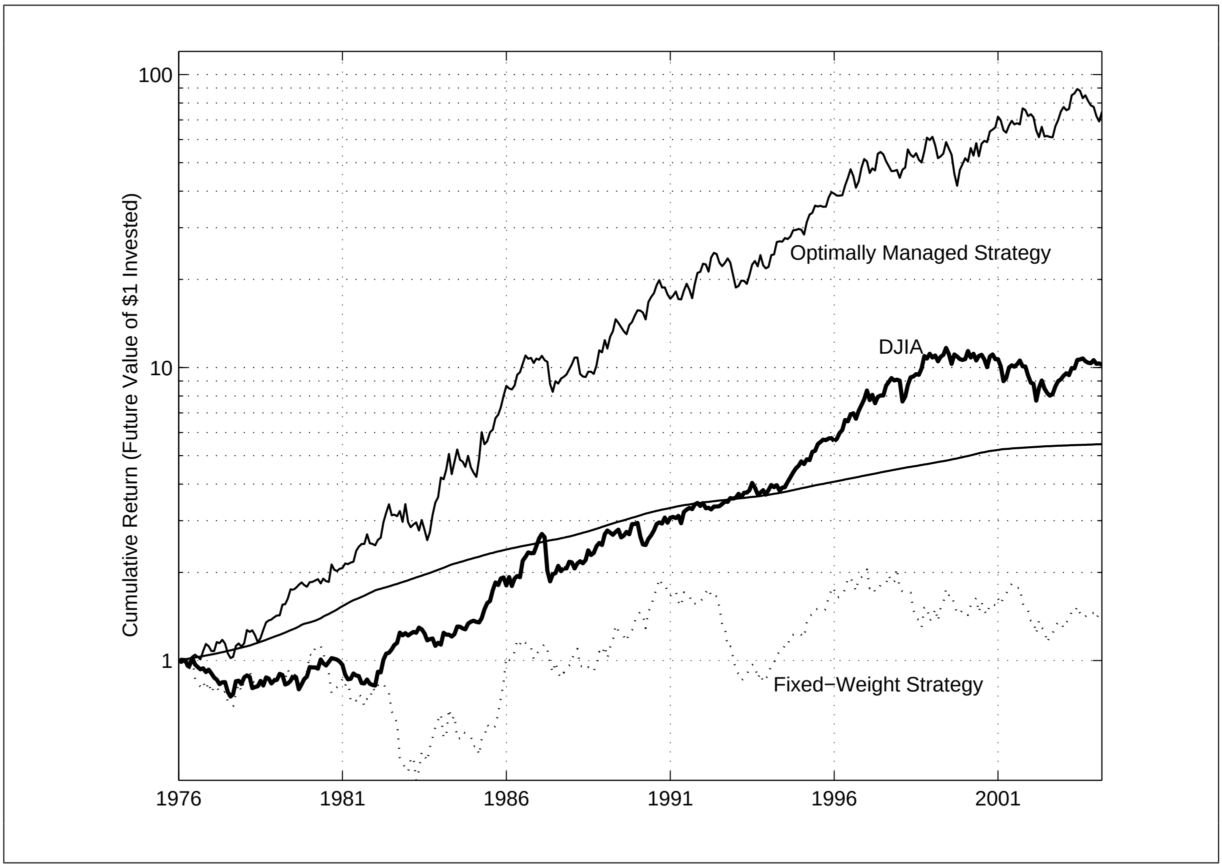


**Figure 1: Cumulative Returns**

This graph shows the cumulative returns from investing one dollar each in the optimally managed maximum-return strategy (solid line), the fixed-weight strategy (dashed line), and the Dow Jones Industrial Average (black line) over the 1976-2004 period. The maximum return strategy has target volatility equal to that of the DJIA over the 1976-2004 period. The weights of the optimal strategy are given in Appendix A, while the fixed-weight strategy is constructed using standard mean-variance theory. The predictive variables used for the optimally managed strategy in this case are term spread (YSPR) and convexity (CONV).

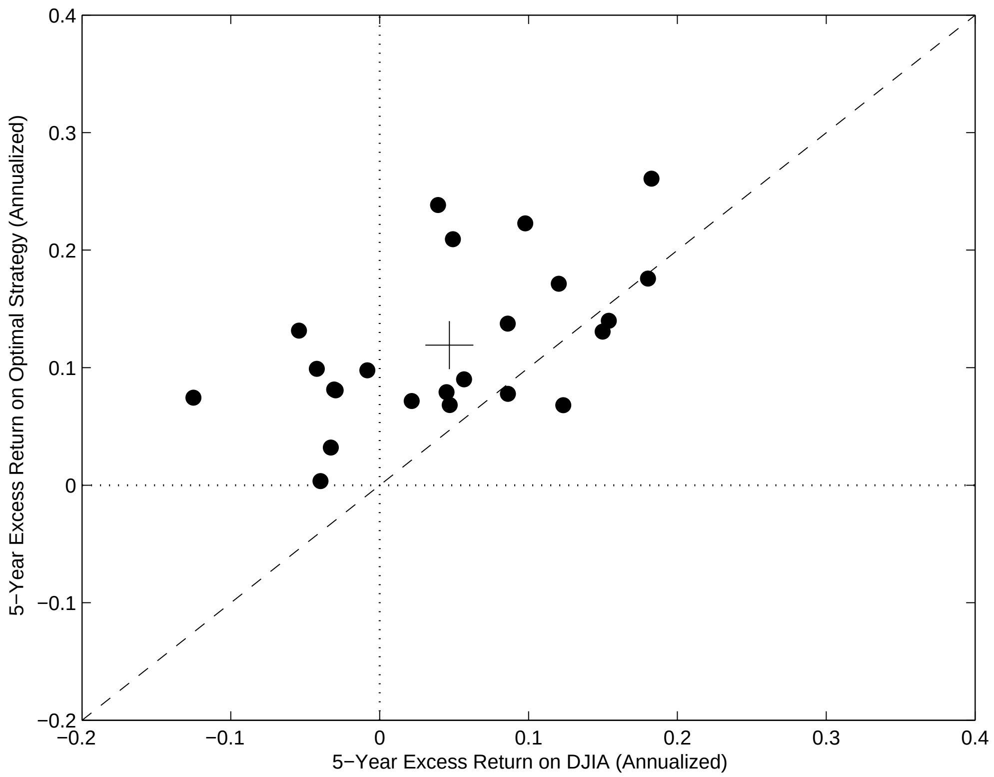


**Figure 2: Five-Year Performance**

This graph plots the five-year performance of the optimally managed strategy against the corresponding performance of the DJIA. Each dot represents the annualized cumulative excess (over T-bill) return over a given five-year period. The return on the DJIA is measured on the horizontal axis, while the return on the optimal strategy is plotted on the vertical axis. The dotted lines correspond to zero excess return (i.e. the strategy returns equal the T-bill return). The cross ('+') marks the average excess returns. The maximum return strategy has target volatility equal to that of the DJIA over the 1976-2004 period. The weights of the optimal strategy are given in Appendix A. The predictive variables used for the optimally managed strategy in this case are term spread (YSPR) and convexity (CONV).