JULIA BRETTSCHNEIDER
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WHAT DOES A MATHEMATICIAN SEE IN A PIECE OF KNITTING?

WHAT DOES A **MATHEMATICIAN** SEE IN A PIECE OF KNITTING?

Topologist: knots and holes

Geometer: curvatures and distances

Number theorist: series of colours or stitches

Probabilist: pseudorandom structure or chaos

Discrete mathematician: algorithms

Statistician: variance and errors

and more…

WHAT IS KNITTING?

COMMON DEFINITION

Making yarn into fabric using needles and suitable hand and finger movements

MATHEMATICAL DEFINITION

???

WHAT IS KNITTING?

COMMON DEFINITION

Making yarn into fabric using needles and suitable hand and finger movements

MATHEMATICAL DEFINITION

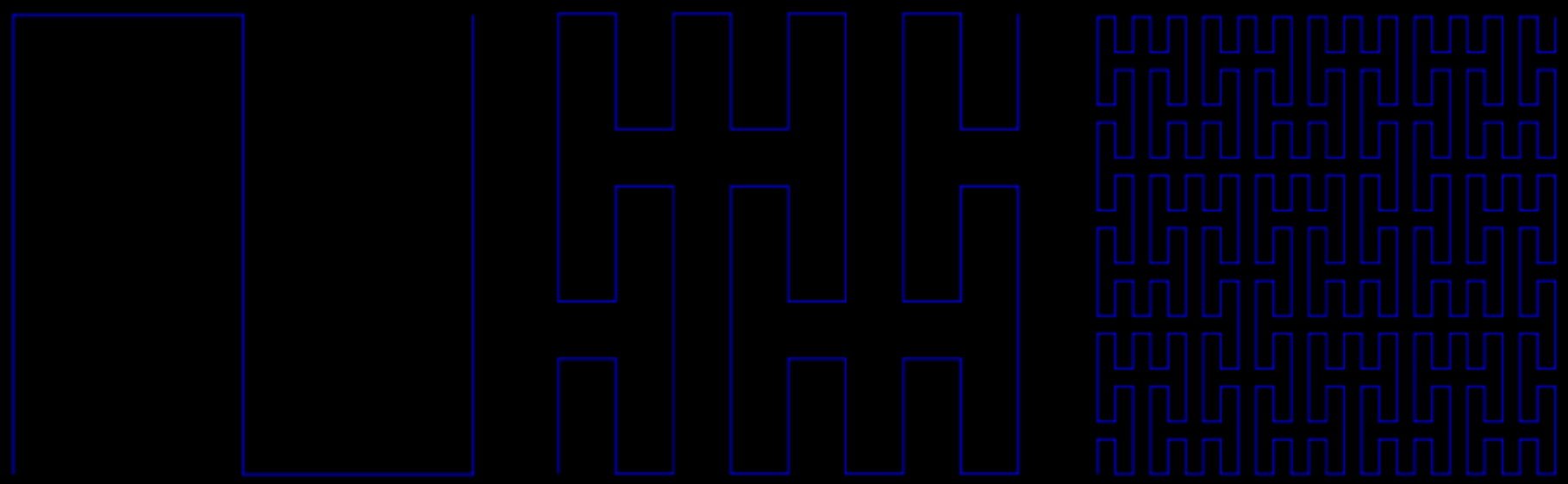
Transforming a one-dimensional object into a two- dimensional object through a composition of knots

SPACE FILLING CURVES

DEFINITION

A space-filling curve is a curve whose range contains the entire two-dimensional unit square (or more generally an n-dimensional hypercube)

EXAMPLE: PEANO CURVE



Peano, G. (1890), "Sur une courbe, qui remplit toute une aire plane", *Mathematische Annalen* **36** (1): 157–160

HOW MANY SIDES?

- Take a long strip of paper. How many sides does this have?
- Turn one of the short sides and glue it to the other.
- Start somewhere and draw a line parallel to the long sides until you come back to the beginning?
- How many sides does this have?

HOW MANY SIDES?

- Take a long strip of paper. How many sides does this have?
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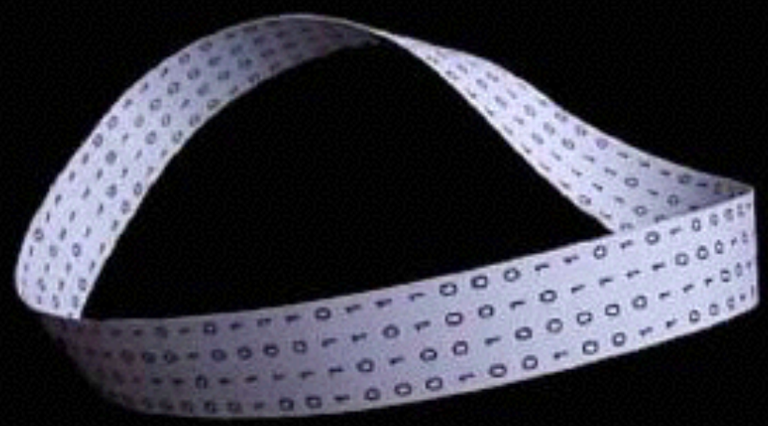
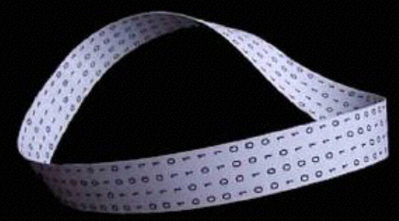
- Start somewhere and draw a line parallel to the long sides until you come back to the beginning?
- How many sides does this have?

MOEBIUS STRIP



PROJECT

KNITTING A MOEBIUS STRIP



TECHNIQUES

- Circular knitting with initial twist
- Knit a rectangle, twist and graft short sides together

LITERATURE

Book by Cat Bordhi

HOW MANY DIMENSIONS?

- Unknitted yarn: 1-dim
- Flat project, e.g. washcloth: 2-dim
- Hat: locally about flat, but as a whole part of space.

2- or 3-dim?

HOW MANY DIMENSIONS?

- Unknitted yarn: 1-dim
- Flat project, e.g. washcloth: 2-dim
- Hat: locally flat, but as a whole part of space,
  hence 2-dim

DEFINITION

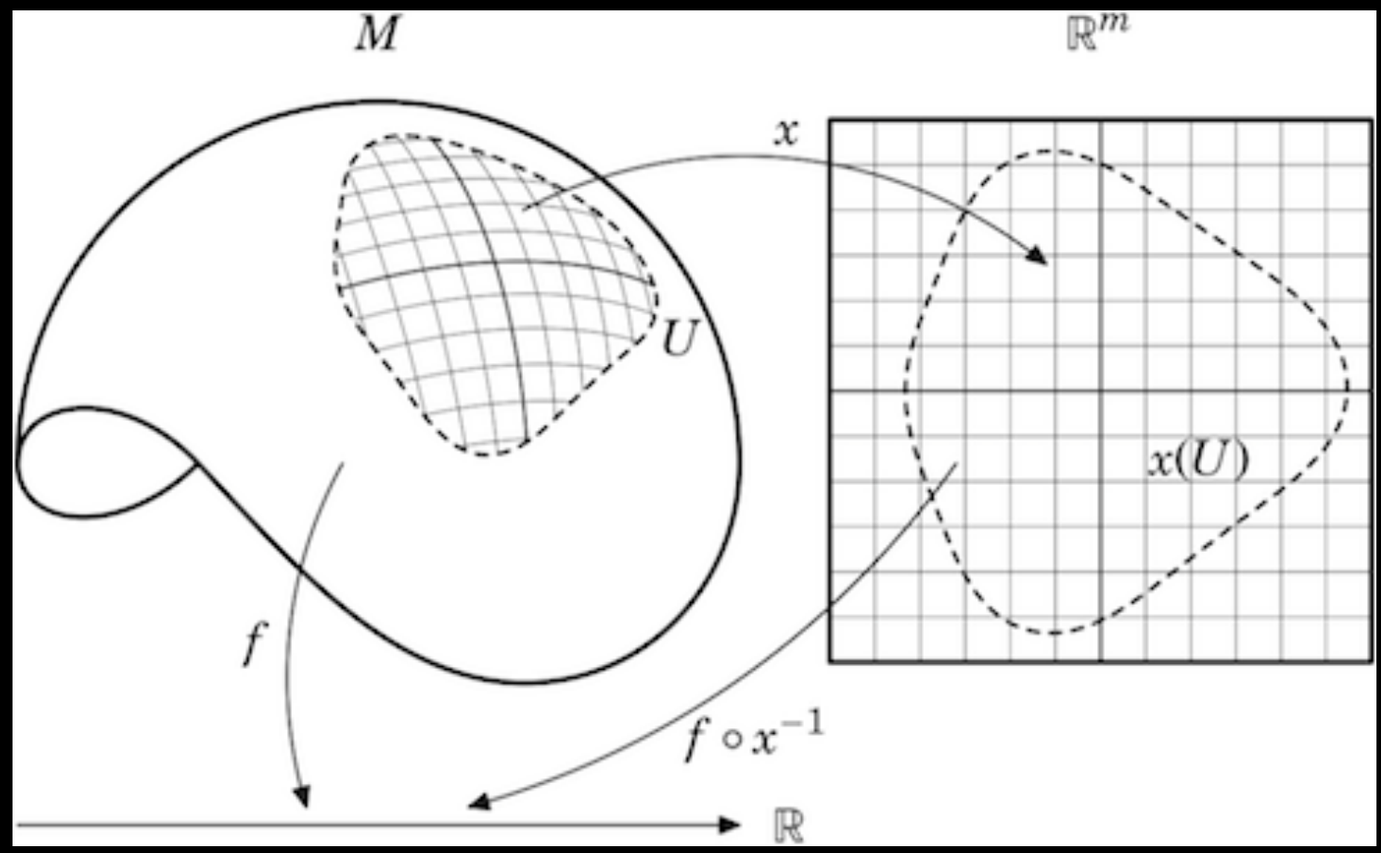
A set of points is called 2-dimensional manifold if it is locally like a plane.

More precisely…

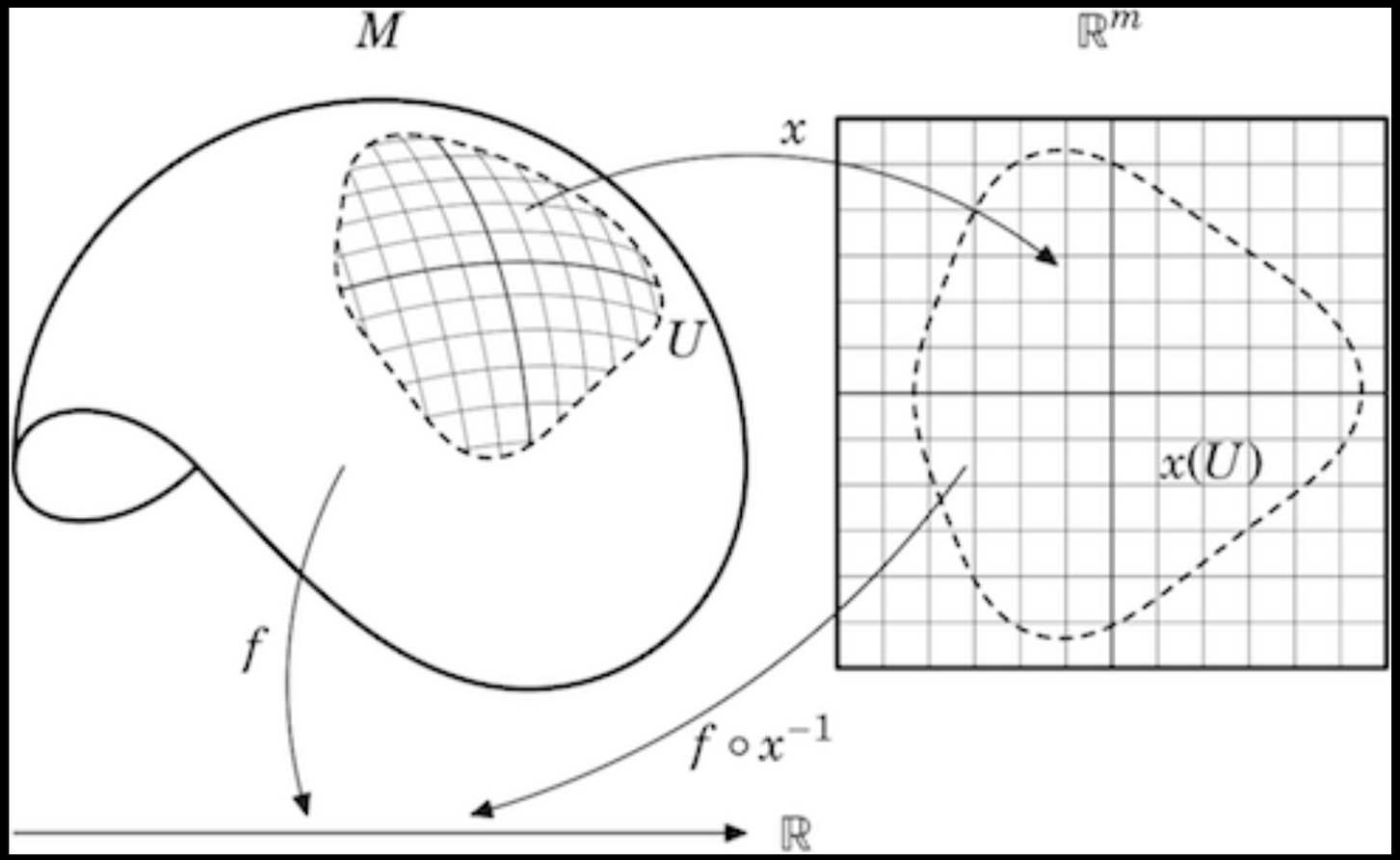
MANIFOLDS

DEFINITION

A set of points is called 2-dimensional manifold if it is locally like a plane. More precisely, each of its points has a neighbourhood that is homeomorphic to the 2-dimensional Euclidean space.



KNITTING MANIFOLDS



A hat is a 2-dimensional manifold

CURVATURE

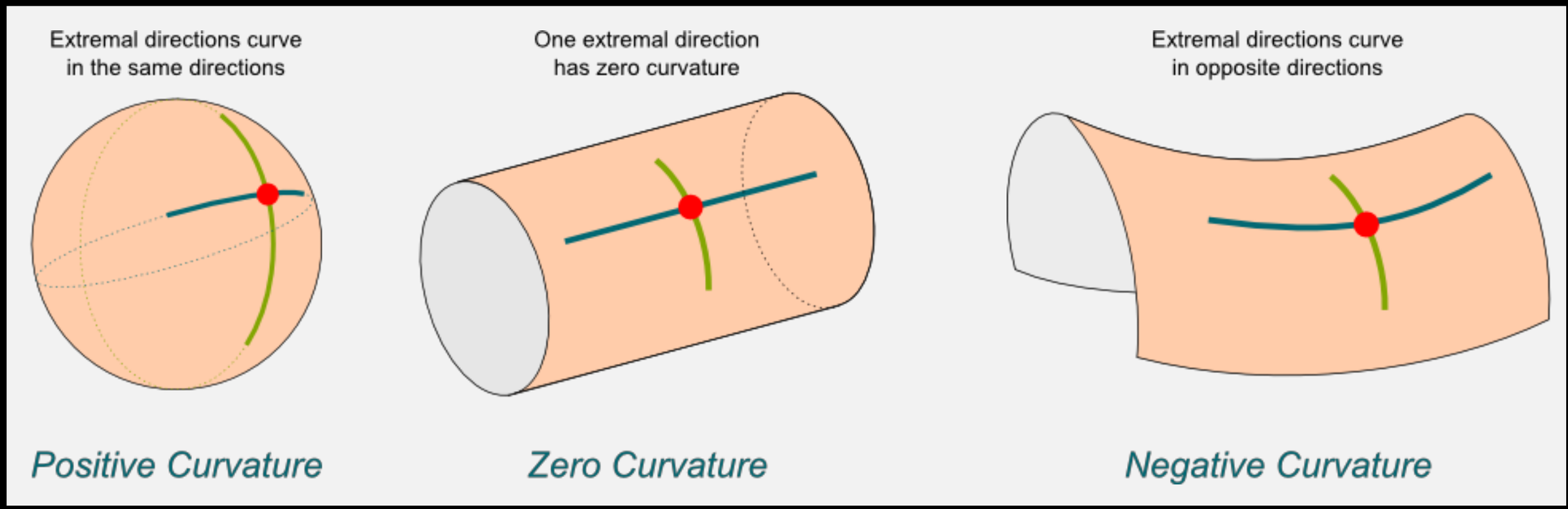
- No curvature: flat, cylinder
- Positive curvature: e.g. ball
- Negative curvature: e.g. crisp

DEFINITION (INFORMAL)

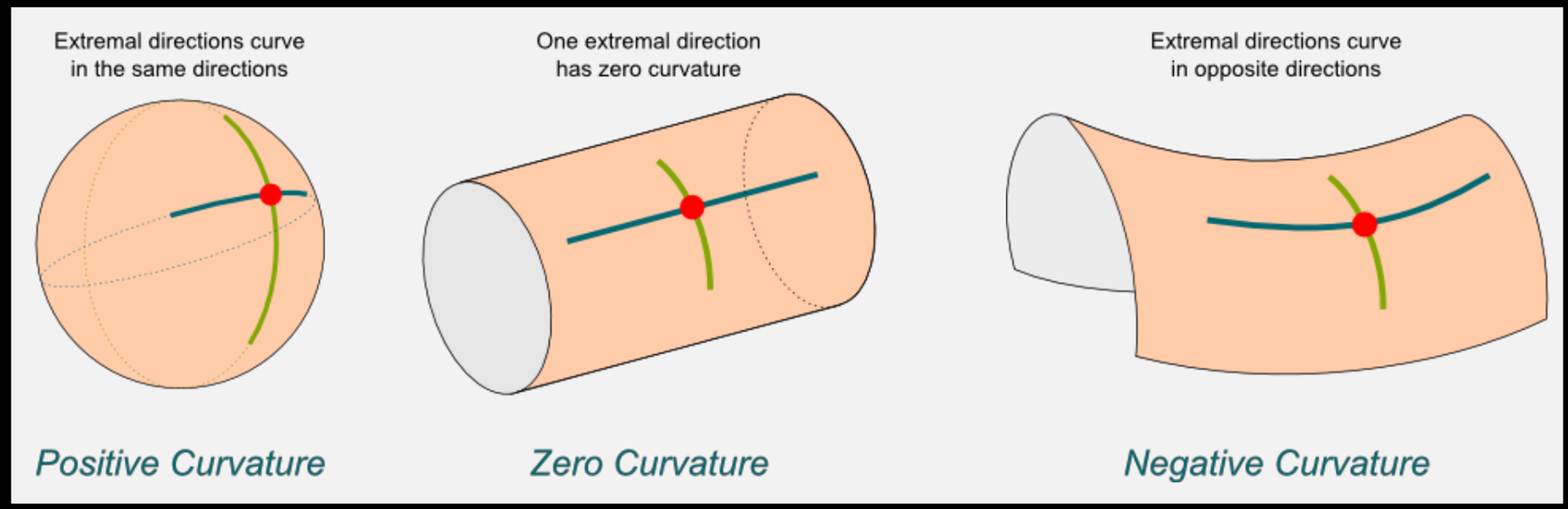
Patch a small flat piece of paper onto the object starting in the centre. If it makes folds the curvature is positive. If it tears the curvature is negative. If it stays intact the curvature is zero.

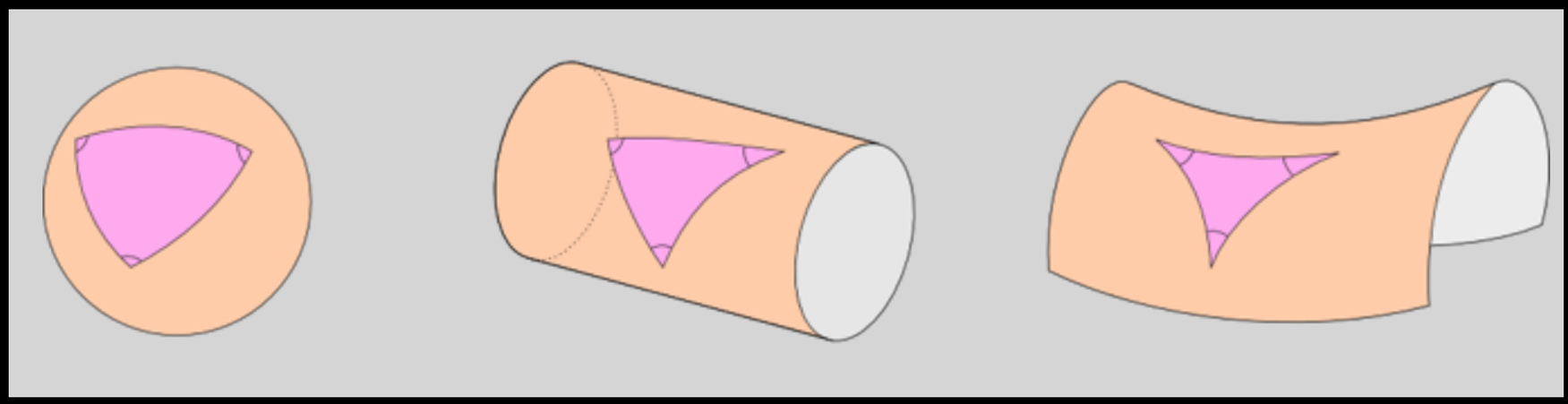
CURVATURE

DEFINITION



CURVATURE





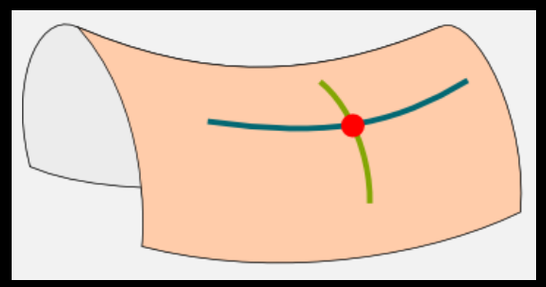
Sum of angles:	>180	=180	<180
Geometry:	Spherical	Euclidean	Hyperbolic

PROJECT

CRAFTING HYPERBOLIC SPACES

TECHNIQUES

- Crochet or knit in a circle
- Start with tiny circle and increase (e.g. add 1 every 3rd stitch)

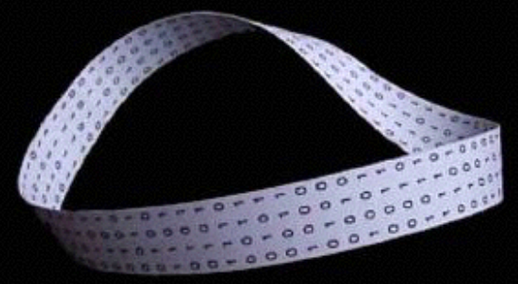


LITERATURE

Book by Daina Taimina

QUESTIONS

What is the curvature of a Moebius strip?



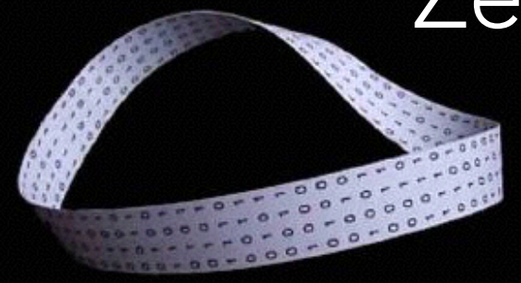
What is the curvature of a torus?

What is the curvature of an egg?

What is the curvature of a fried egg?

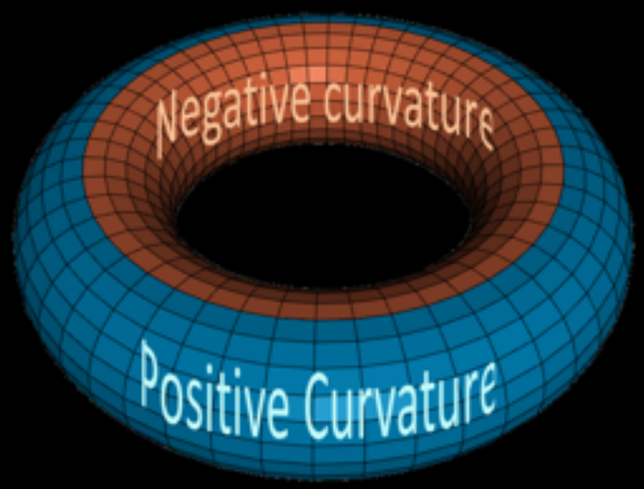
QUESTIONS

What is the curvature of a Moebius strip?



Zero

What is the curvature of a torus?



What is the curvature of an egg?

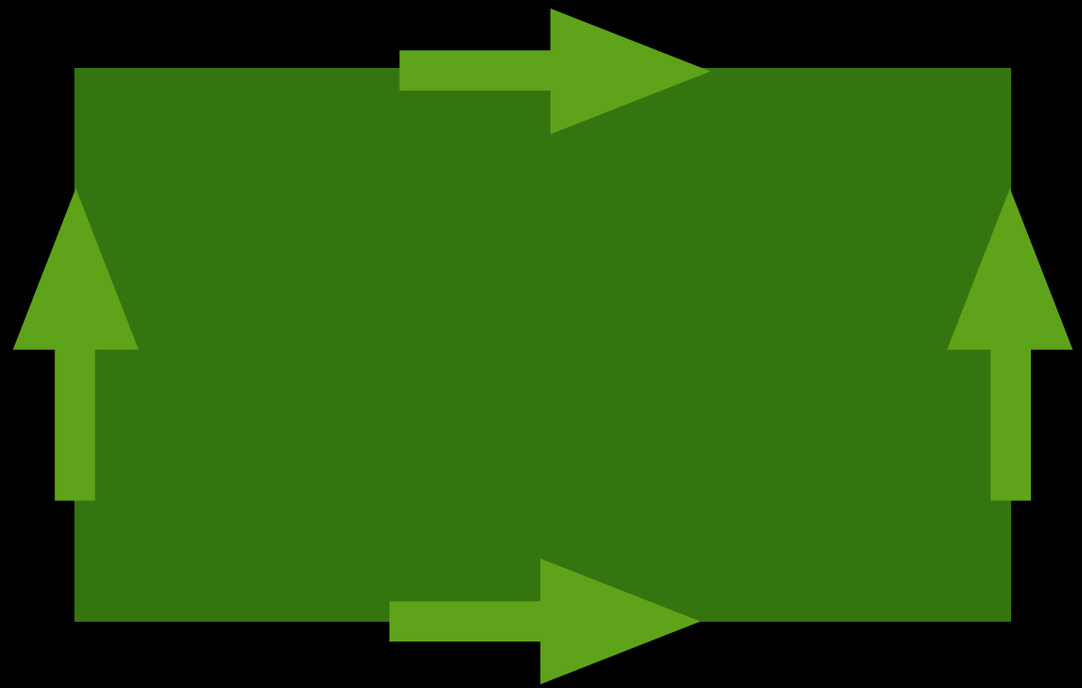
Positive

What is the curvature of a fried egg?

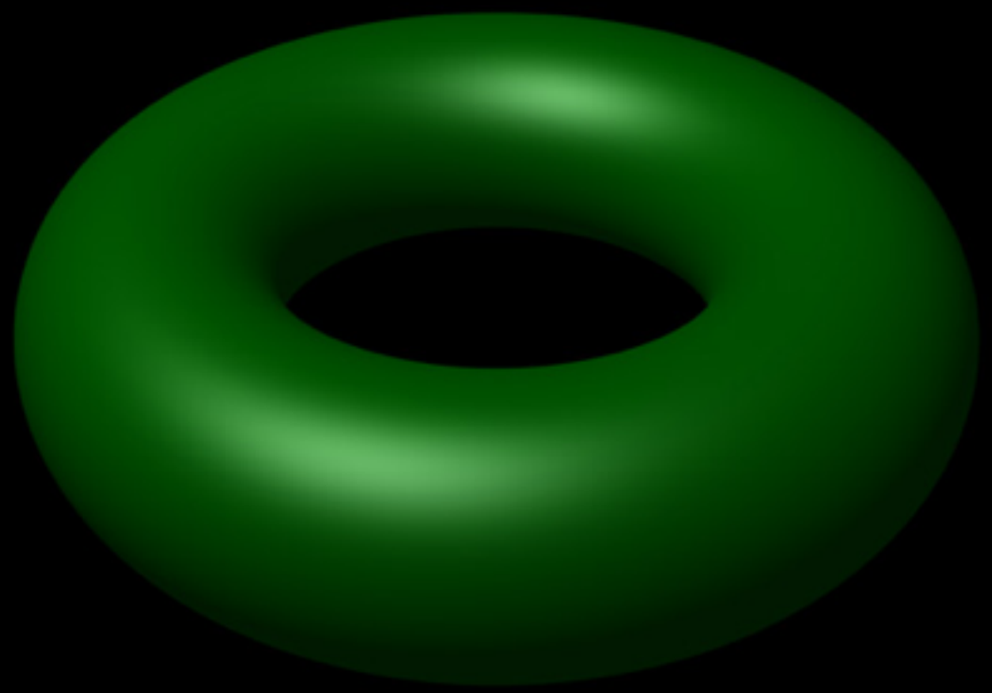
Positive/ zero

TORUS CONSTRUCTION

- Take a rectangle
- Glue top and bottom side
- Glue the other opposite sides

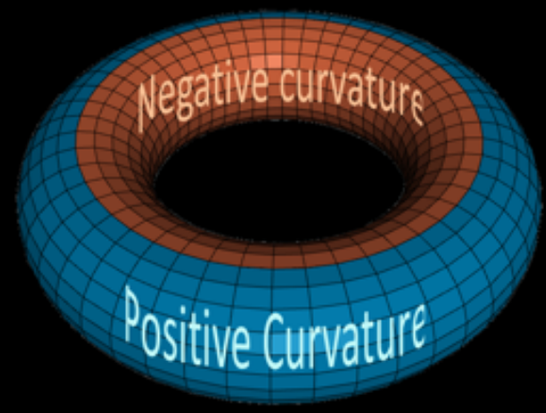


Works better with rubber than with paper.



PROJECT

KNITTING A TORUS



TECHNIQUES

- Work in rows (knit and purl)
- Use shorter rows to make outer diameter smaller than inner diameter

LITERATURE

Book by Sarah-Maria Belcastro & Carolyn Yackel

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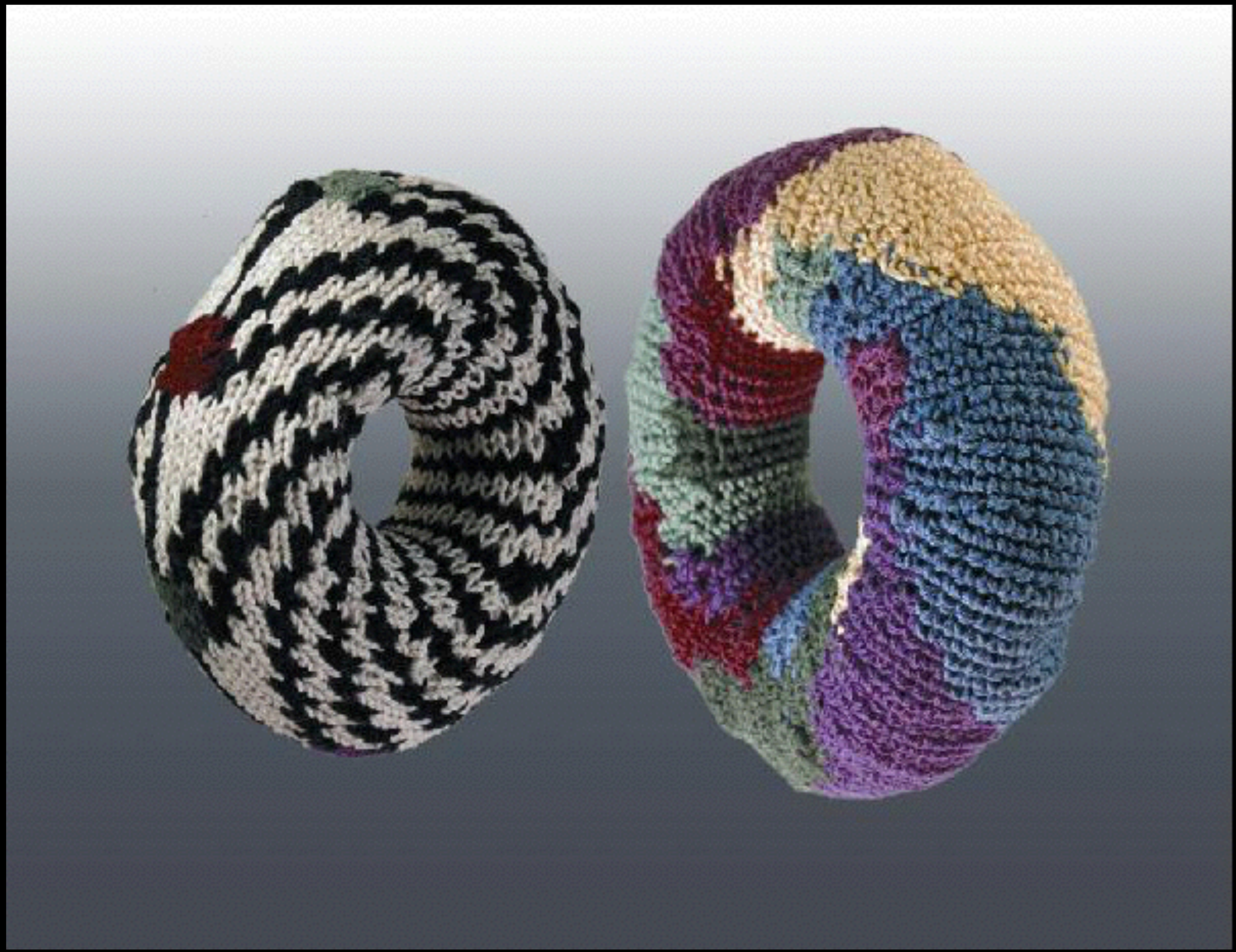
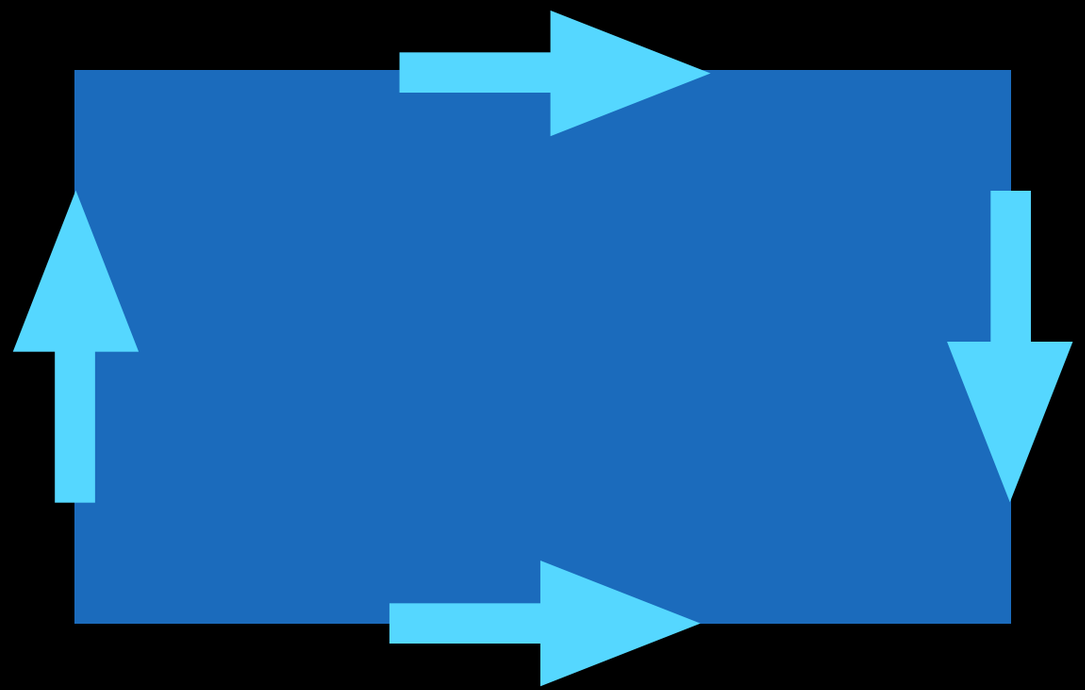


KLEIN BOTTLE CONSTRUCTION

- Take a rectangle
- Glue top and bottom side
- Glue the other opposite sides with a twist



Works better with rubber than with paper.

http://www.kleinbottle.com

FURTHER RESOURCES

Home of mathematical knitting:
http://www.toroidalsnark.net/mathknit.html

http://www.americanscientist.org/issues/pub/adventures-in-mathematical-knitting

http://www.woollythoughts.com/index.html

http://www.math.cornell.edu/~dtaimina/