ES2A7 - Fluid Mechanics Example Classes

Model Answers to Example Questions (Set III)

<u>Question 1:</u> Distance between molecules:

Find the mean distance between water molecules
- in a liquid state
- in vaporous state at T=400K and P=1bar. This gas is assumed to follow the law of perfect gas.

Data:
- Density of liquid water: $\rho = 1000\ \text{kg.m}^{-3}$
- Avogadro Number: $N_A = 6.10^{23}$ molecules.mol^{-1}
- Molecular mass of water: $M = 18\ \text{g.mol}^{-1}$
- Perfect Gas constant: $R = 8.31\ \text{J.K}^{-1}.\text{mol}^{-1}$

Liquid Water :

There are $N = \frac{\rho}{M} N_A$ particles in a volume of $1m^3$.

The mean distance between 2 particles is thus :

$$d = \sqrt[3]{\frac{M}{\rho N_A}} = \sqrt[3]{\frac{18.10^{-3}}{10^3 \times 6.10^{23}}} = \sqrt[3]{30.10^{-30}} = 3.10^{-10}m$$

Gas Water :

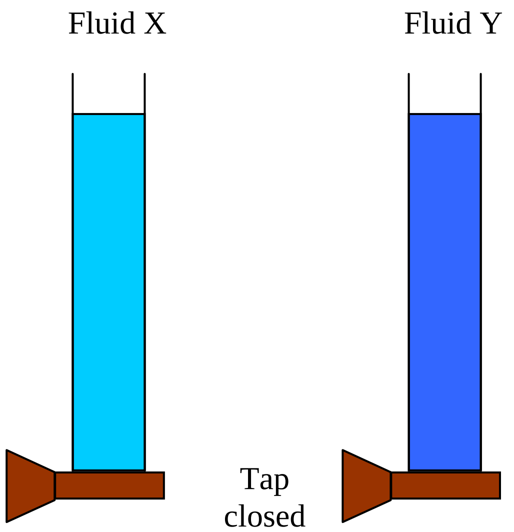
The vapour being considered as a perfect gas, N_A molecules fill up a volume $v = \frac{RT}{P}$

Thus the mean distance between 2 particles is :

$$d = \sqrt[3]{\frac{RT}{PN_A}} = \sqrt[3]{\frac{8.31 \times 400}{10^5 \times 6.10^{23}}} = \sqrt[3]{55.10^{-27}} = 40.10^{-10}m$$

Question 2: Type of Fluid

Let consider 2 identical vertical tubes (X and Y) with a free surface on their upper part and close by a tap on their lower part. Both are filled with the same height of fluid: A Newtonian fluid is used with tube X whereas a non-Newtonian fluid is used with tube Y.
Taps are open simultaneously. At the beginning the fluid in tube Y flows more rapidly than that in tube X. After a moment, when the height of fluid becomes small, the non-Newtonian fluid decelerates compare to the Newtonian fluid, and in the end the Newtonian fluid flows fast than the non-Newtonian fluid.
What is the type of the non-Newtonian fluid ? Explain the experiment and give an example of such fluid



Sketch of the experiment before it starts

The fluid Y is Pseudo-Plastic (or shearthinning) fluid. Its viscosity is smaller under large shear stress. At the beginning of the experiment, the weight of fluid is maximal and the pressure on the fluid close to the tap is important. As one goes along, the quantity of water inside the tube decreases as well as the pressure. The stress is so less important and the viscosity of the fluid Y increases and becomes, in that particular case, larger than the viscosity of the fluid X. This latter is constant since X is a Newtonian fluid. The Y flow rate becomes smaller because of the viscous forces are stronger.

Paints are generally Pseudo-Plastic fluid, they don’t drop on the wall unless you apply a stress on it thank to your paintbrush.

<u>Question 3:</u> Type of Fluid

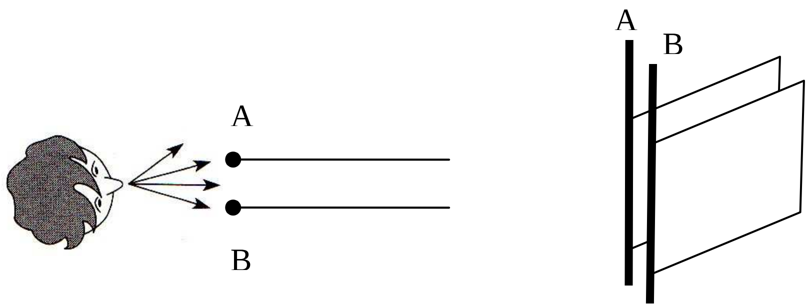
Classify the following fluid: Ketchup, Silly putty, mixture of water and corn flour, honey, wet sand, blood, paint

Pseudo-plastic ; Paint, blood, ketchup
Dilatant : Wet sand, putty, mixture of water and corn flour
Newtonian : Honey

Thixotropic: Ketchup

<u>Question 4:</u> Venturi Effect

Let consider two vertical pieces of paper (typical $80g/m^2$ A4) separate by a small distance (2-3cm). On one side, the vertical edges of both sheet of paper are hold firmly so that they can't move. The other edges remain free of motion.
Describe what happen when someone standing behind the fixed edges blows between the 2 sheets of paper. Explain the experiment.



<u>Picture at t=0. A and B are fixed rod holding the edges of paper sheets fix</u>

The 2 sheets of papers are attracted to one another and get in contact. Indeed as there is an air flow between the sheet of the paper, the pressure has to be smaller than on the other side of the sheet where the air is at rest. This comes from the Bernoulli equation applies between 2 points located on each side of one of the sheets of paper at the same height.

<u>Question 5:</u> Newtonian Fluid

Let consider the flow within a Taylor-Couette device. It consists of two concentric cylinder of which inner cylinder rotates at Ω rad/s. At low rotation speed, the velocity field is purely azimuthal and evolves as $v_\theta(r) = Xr + Y/r$ for a Newtonian fluid (the effect of the boundary in the axial direction are neglected). Find the expression of X and Y as a function of the radii of the cylinders, and then determine the expression of the mean velocity.

The velocity field has to verify the no slip condition on its boundaries on solid surface. The velocity should then verify: $V_\theta(R_i) = \Omega R_i$ and $V_\theta(R_o) = 0$ where R_i and R_o are respectively the radii of the inner and the outer cylinder.
To determine X and Y, one should then solve the system:

$$\begin{cases} \Omega R_i = X R_i + \dfrac{Y}{R_i} \\ 0 = X R_o + \dfrac{Y}{R_o} \end{cases}$$

$$\Leftrightarrow \begin{cases} \Omega R_i^2 = X R_i^2 - X R_o^2 \\ Y = -X R_o^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} X = \Omega R_i^2 \big/ \left(R_i^2 - R_o^2\right) \\ Y = -\Omega R_i^2 R_o^2 \big/ \left(R_i^2 - R_o^2\right) \end{cases}$$

The mean velocity is determined by integrating the velocity profile between the 2 cylinders:

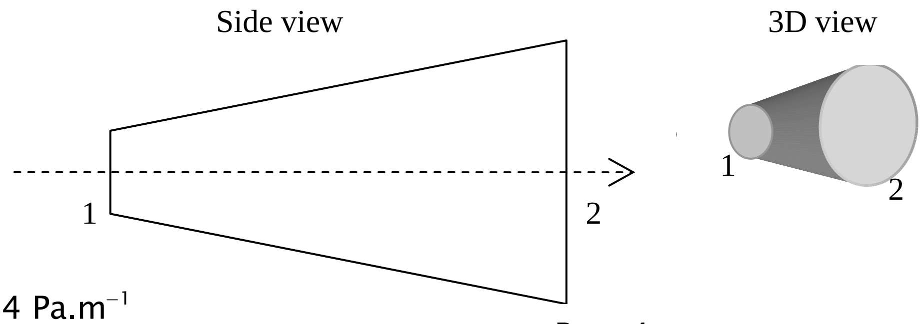
$$\overline{V} = \frac{1}{R_o - R_i} \int_{R_i}^{R_o} V_\theta(r)\, dr = \frac{1}{R_o - R_i} \left[\frac{Xr^2}{2} - \frac{Y}{r^2} \right]_{R_i}^{R_o}$$

$$\overline{V} = \frac{1}{R_o - R_i} \left(\frac{X\left(R_o^{\,2} - R_i^{\,2}\right)}{2} + \frac{Y}{R_i^{\,2}} - \frac{Y}{R_o^{\,2}} \right)$$

$$\overline{V} = \frac{\Omega}{R_o - R_i} \left(1 - \frac{R_i^2}{2} \right)$$

Question 6: Newtonian Fluid

Let consider the water flow through an expanding tube (dynamic viscosity $\mu = 8.9 \times 10^{-4} \text{Pa.s}$). As a Newtonian fluid, the velocity of the water is dependant on its radial position within the tube. The radial variation of the velocity is given by the following relation: $\dfrac{dV(r)}{dr} = \dfrac{r}{2\mu} \dfrac{dP}{dx}$.



$R_1 = 1.5\ \text{cm}$

$\left(\dfrac{dP}{dx}\right)_1 = -7.64\ \text{Pa.m}^{-1}$

$R_2 = 4\ \text{cm}$

Determine the mean velocity of the water flow at the exit (section 2).
(Hint: Calculate first the velocity profile V(r) and then the mean velocity at the entrance (section 1)).

Velocity profile is obtained by integrating the velocity gradient:

$$V(r) = \int \frac{r}{2\mu}\frac{dP}{dx}dr = \frac{r^2}{4\mu}\frac{dP}{dx} + cte$$

At the boundary (r=R), the velocity has to be zero to verify the no-slip condition. So:

$$V(r) = \frac{r^2 - R^2}{4\mu}\frac{dP}{dx}$$

The mean velocity is found by integrating the velocity profile in the radial direction:

$$\overline{V} = \frac{1}{R}\int_0^R V(r)\,dr = \frac{1}{4R\mu}\frac{dP}{dx}\left[\frac{r^3}{3} - R^2 r\right]_0^R = -\frac{R^2}{6\mu}\frac{dP}{dx}$$

At the entrance:

$$\overline{V_1} = \frac{0.015}{6\times 8.9\times 10^{-4}}\times 7.64 = 21.5\text{m.s}^{-1}$$

Finally the mean velocity at the exit is determined using the mass conservation relation in the case of a non-compressible fluid in a stationary flow:

$$\rho A_1 \overline{V_1} = \rho A_2 \overline{V_2}$$

$$\Leftrightarrow \overline{V_2} = \frac{A_1}{A_2}\overline{V_1} = \frac{R_1^2}{R_2^2}\overline{V_1}$$

$$\Leftrightarrow \overline{V_2} = \frac{1.5^2}{4^2}\times 21.5 = 3\text{m.s}^{-1}$$