Localised Collective Excitations in Doped Graphene in a Strong Magnetic Field

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Summary [arXiv:0902.4176]

We consider collective excitations in graphene in a strong perpendicular magnetic field with a single Coulomb impurity. We show that localised collective modes split off the magnetoplasmon continuum and in addition, quasibound states are formed within the continuum. A study of the evolution of the dipole strengths and energies of magneto-optical transitions is performed for integer filling factors $\nu = 1, 2, 3, 4$ of the zeroth Landau level (LL). We predict impurity absorption peaks above as well as below the cyclotron resonance. In contrast to the 2D electron gas (2DEG) there are strong peaks for both circular polarisations of light, whose dipole strengths increase linearly with magnetic field. We find that the single-particle electron-hole symmetry of graphene leads to a duality between the spectra of collective modes for the positively charged donor impurity, D^+, and the negatively charged acceptor impurity, A^-. The duality shows up as a set of complementary magneto-absorption peaks, which are active in different circular polarizations.

Single particle in a magnetic field

- Single impurity, so use axially symmetric gauge: $\mathbf{A} = \frac{1}{2}\mathrm{B} \times \mathbf{r}$
- Wavefunction for single electron in K valley (pseudospin $\Uparrow$):

$$\Phi_{nms\Uparrow}(\mathbf{r}) = \langle \mathbf{r}|c^\dagger_{nms\Uparrow}|0\rangle = 2^{\frac{1}{2}(\delta_{n,0}-1)} \begin{pmatrix} \mathrm{sign}(n)\phi_{|n|-1\,m}(\mathbf{r}) \\ \phi_{|n|\,m}(\mathbf{r}) \\ 0 \\ 0 \end{pmatrix} \chi_s\,, \tag{4}$$

where $\phi_{|n|\,m}(\mathbf{r})$ is regular LL wavefunction for a 2DEG and χ_s is spin part.

- Corresponding energy: $\epsilon_{\mathcal{N}} = \mathrm{sign}(n)\hbar\omega_c\sqrt{|n|} + \hbar\omega_s s_z + \hbar\omega_\nu\sigma_z$,

where $\hbar\omega_c = v_F\sqrt{2e\hbar B}$ is cyclotron energy in graphene and other terms represent spin and valley splitting.

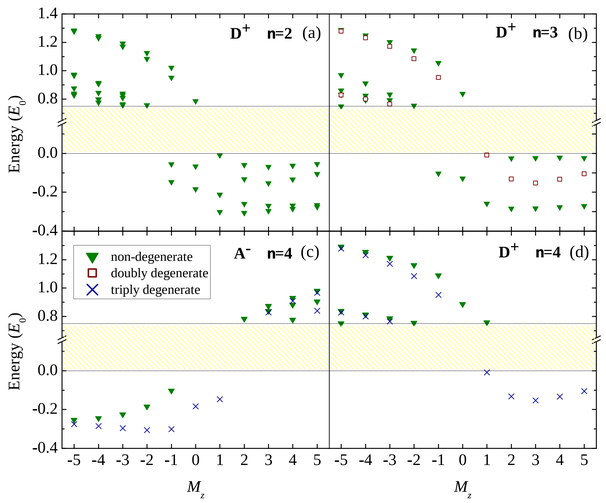

Figure 1: *Energies of magnetoplasmons bound on an impurity versus projection of the orbital angular momentum M_z for (a) a donor impurity and filling factor $\nu = 2$, (b) a donor impurity and $\nu = 3$, (c) an acceptor impurity and $\nu = 4$ and (d) a donor impurity and $\nu = 4$. The shaded area represents the band.*

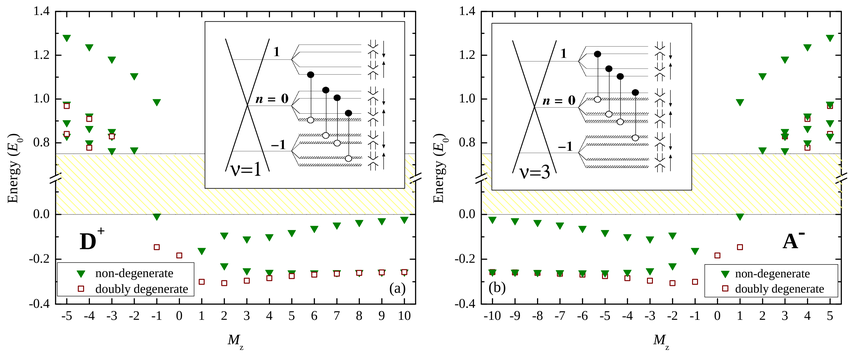

Figure 2: *Energies of magnetoplasmons bound on an impurity versus projection of the orbital angular momentum M_z for (a) a donor impurity and filling factor $\nu = 1$, and (b) an acceptor impurity and $\nu = 3$. The spectra exhibit the symmetry $D^+ \leftrightarrow A^-$, $M_z \leftrightarrow -M_z$ and $\nu \leftrightarrow 4 - \nu$. The hatched area of width $0.75E_0$ represents the continuum of extended magnetoplasmons.*

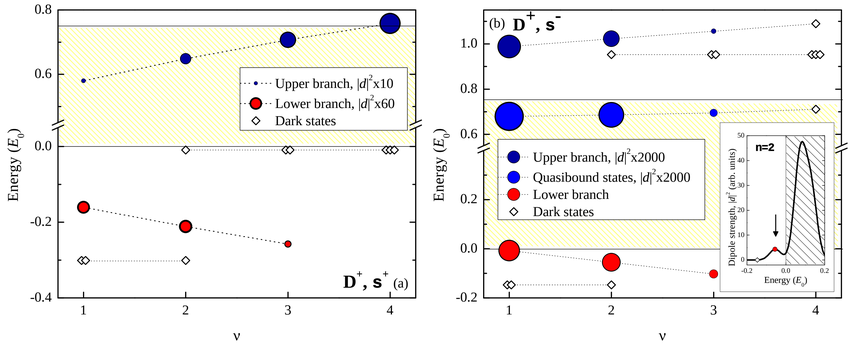

Figure 3: *Evolution with filling factor ν of energies and optical strengths of magnetoplasmons bound on the D^+ with (a) $M_z = 1$ active in the σ^+ polarization and (b) with $M_z = -1$ active in the σ^- polarization. Optically active states are indicated by circles with sizes $\sim |d^\pm_\nu|^2$. Inset: Dipole strength $|d^-_\nu|^2$ vs energy for $\nu = 2$. The spectra were convoluted with a Gaussian of width $0.03E_0$. The arrow indicates an impurity-related feature below $\hbar\widetilde{\omega}_c$ (below energy zero in the Figure).*

Results

- Energies in units of $E_0 = (\pi/2)^{1/2}e^2/\varepsilon l_B$ (l_B is magnetic length)
- Each LL has four sublevels due to spin and pseudospin splitting. We compute results for sublevel filling factors $\nu = 1, 2, 3, 4$ of the $n = 0$ LL. In general infinitely many excitations with the same M_z are mixed. We focus on mixing between excitations with the same single particle cyclotron energies which involve the $n = 0$ LL as an initial or final state. There are four such excitations with no spin or pseudospin flip (see insets in Fig. 2). We present results for these excitations only, since they alone are electrically dipole active.
- The electron-hole symmetry of graphene leads to a duality in the bound states: For sublevels $\nu = 1, 2, 3$ of LL with number n, the eigenstates and eigenenergies of excitations are identical to those for sublevels of LL with number $-n$ under the following:

$$M_z \leftrightarrow -M_z$$
$$\nu \leftrightarrow \nu - 4$$
$$D^+ \leftrightarrow A^-.$$

Theory

- In graphene Kohn's theorem doesn't apply and interactions are very important. Need to consider collective excitations:

$$Q^\dagger_{\mathcal{N}_1\mathcal{N}_2 M_z} = \sum_{m_1,m_2=0}^{\infty} A_{\mathcal{N}_1\mathcal{N}_2 M_z}(m_1, m_2) c^\dagger_{\mathcal{N}_1 m_1} d^\dagger_{\mathcal{N}_2 m_2}\,, \tag{1}$$

where $c^\dagger_{\mathcal{N}_1 m_1}$ and $d^\dagger_{\mathcal{N}_2 m_2}$ are creation operators for electron and hole, $m = 0, 1, \ldots$ is oscillator quantum number and $\mathcal{N} = ns\sigma$ represents LL index $n \in \mathbb{Z}$, spin $s = \uparrow, \downarrow$ and valley pseudospin $\sigma = \Uparrow, \Downarrow$.

- Excitations labelled by M_z, the z component of orbital angular momentum, such that expansion coefficients satisfy $A_{\mathcal{N}_1\mathcal{N}_2 M_z}(m_1, m_2) \sim \delta_{M_z,|n_1|-m_1-|n_2|+m_2}$.
- $Q^\dagger_{\mathcal{N}_1\mathcal{N}_2 M_z}$ acts on $|\nu\rangle$, the ground state with integer filling factor ν.
- Hamiltonian is

$$\begin{aligned} \hat{H}^{\mathcal{N}_1'\mathcal{N}_2'}_{\mathcal{N}_1\mathcal{N}_2} = &\sum_{m=0}^{\infty} (\bar{\epsilon}_{\mathcal{N}_1} + \mathcal{V}_{\mathcal{N}_1 m}) c^\dagger_{\mathcal{N}_1' m} c_{\mathcal{N}_1 m} \\ &- \sum_{m=0}^{\infty} (\bar{\epsilon}_{\mathcal{N}_2} + \mathcal{V}_{\mathcal{N}_2 m}) d^\dagger_{\mathcal{N}_2' m} d_{\mathcal{N}_2 m} \\ &+ \sum_{\substack{m_1,m_2 \\ m_1',m_2'}} \left(\mathcal{W}^{\mathcal{N}_2 m_2 \mathcal{N}_1' m_1'}_{\mathcal{N}_1 m_1 \mathcal{N}_2' m_2'} - \mathcal{W}^{\mathcal{N}_1' m_1' \mathcal{N}_2 m_2}_{\mathcal{N}_1 m_1 \mathcal{N}_2' m_2'} \right) c^\dagger_{\mathcal{N}_1' m_1'} d^\dagger_{\mathcal{N}_2' m_2'} d_{\mathcal{N}_2 m_2} c_{\mathcal{N}_1 m_1}\,. \end{aligned} \tag{2}$$

$\bar{\epsilon}_{\mathcal{N}}$ is single particle LL energy renormalised by self energy corrections due to exchange with electrons in lower cone, $\mathcal{V}_{\mathcal{N}_1 m} = \langle \phi_{nm}|V(r)|\phi_{nm}\rangle$ is impurity matrix element, $\mathcal{W}^{\mathcal{N}_1' m_1' \mathcal{N}_2 m_2}_{\mathcal{N}_1 m_1 \mathcal{N}_2' m_2'} = \langle \Phi_{\mathcal{N}_1' m_1'} \Phi_{\mathcal{N}_2' m_2'} | U_{ee} | \Phi_{\mathcal{N}_1 m_1} \Phi_{\mathcal{N}_2 m_2} \rangle = \delta_{s_1,s_1'}\delta_{\sigma_1,\sigma_1'}\delta_{s_2,s_2'}\delta_{\sigma_2,\sigma_2'}\mathcal{U}^{n_1' m_1' n_2' m_2'}_{n_1 m_1 n_2 m_2}$ is direct electron-hole attraction and $\mathcal{W}^{\mathcal{N}_2 m_2 \mathcal{N}_1' m_1'}_{\mathcal{N}_1 m_1 \mathcal{N}_2' m_2'}$ is repulsive electron-hole exchange.

- Graphene matrix elements, $\mathcal{U}^{n_1' m_1' n_2' m_2'}_{n_1 m_1 n_2 m_2}$, are related to those for 2DEG, $\mathcal{U}^{n_1' m_1' n_2' m_2'}_{n_1 m_1 n_2 m_2}$

$$\begin{aligned} \mathcal{U}^{n_1' m_1' n_2' m_2'}_{n_1 m_1 n_2 m_2} = a_{n_1} a_{n_2} a_{n_1'} a_{n_2'} \Big[& U^{|n_1'|\,m_1'\;|n_2'|\,m_2'}_{|n_1|\,m_1\;|n_2|\,m_2} \\ &+ s_{n_1} s_{n_1'} U^{|n_1'|-1\,m_1'\;|n_2'|\,m_2'}_{|n_1|-1\,m_1\;|n_2|\,m_2} + s_{n_2} s_{n_2'} U^{|n_1'|\,m_1'\;|n_2'|-1\,m_2'}_{|n_1|\,m_1\;|n_2|-1\,m_2} \\ &+ s_{n_1} s_{n_2} s_{n_1'} s_{n_2'} U^{|n_1'|-1\,m_1'\;|n_2'|-1\,m_2'}_{|n_1|-1\,m_1\;|n_2|-1\,m_2} \Big], \end{aligned} \tag{3}$$

Optics

- Interaction of electrons in graphene with light of frequency ω and left (+) and right (−) circular polarizations is described by Hamiltonian

$$\delta H_\pm = \frac{e v_F \mathcal{E}}{i\omega} \begin{pmatrix} \sigma_\pm & 0 \\ 0 & \sigma_\mp \end{pmatrix}, \tag{5}$$

where $\mathcal{E}$ is electric field amplitude and $\sigma_\pm = \sigma_x \pm i\sigma_y$.

- Optical selection rules determined from dipole strength $|d^\pm_\nu|^2 = |\langle M_z = \pm 1|\delta H_\pm|\nu\rangle|^2$, where $|M_z = \pm 1\rangle$ is collective excitation and $|\nu\rangle$ is ground state with filling factor ν.
- Optical selection rules are no spin- or pseudospin flips, $M_z = \pm 1$ and $|n_1| - |n_2| = \pm 1$.

References

2DEG:

[1] A. Dzyubenko and Yu. E. Lozovik, Zh. Eksp. Teor. Fiz. **104**, 3416 (1993).

[2] C. Kallin and B.I. Halperin, Phys. Rev. B **31**, 3635 (1985).

Graphene:

[3] A. Iyengar, J. Wang, H. A. Fertig, and L. Brey, Phys. Rev. B **75**, 125430 (2007).

[4] Yu. A. Bychkov and G. Martinez, Phys. Rev. B **77**, 125417 (2008).